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ITERATION PROCESS WITH ERRORS FOR LOCAL STRONGLY H-ACCRETIVE TYPE MAPPINGS

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Abstract. Some iteration processes of Mann and Ishikawa type with error has been discussed to approximate solution of equation $Tx = f$, where T is locally strongly H - accretive mapping [18] on uniformly smooth Banach space X. This extends an earlier result of Liu [9] on iterative processes with errors. We also extend a result of Weng [20] on iterative processes of dissipative type mappings.

Key Words and Phrases: Mann iteration process, Ishikawa iteration process, strictly pseudo-contractive map, local strongly H-accretive map, accretive map,strongly accretive map.

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1. Introduction

In recent literature interests have been generated to deal with iteration processes which approximates fixed points of nonlinear mappings in a Banach Space with special emphasis on Mann and Ishikawa type of processes. In [10] Liu extended these ideas to deal with Mann and Ishikawa type of processes with errors.

Browder [1] and Kato [8] have introduced the concept of accretive operators to establish that the initial value problem:

$$\frac{du}{dt} + Tu = 0, \quad u(0) = u_0$$

is solvable if T is locally Lipschitzian and m-accretive [3,6], strongly accretive [2,14] and continuous accretive [11,15,16] operators. In [9] Liu dealt with strongly accretive operators, and proved that when E is a uniformly smooth Banach space and $T : K \to K$ is a strongly accretive mapping where K is a nonempty closed convex and bounded subset of E then both Mann and Ishikawa iteration with errors could be used to approximate the unique solution of the equation $Tx = f$. We extend this result of Liu to cover a new class called local strongly H-accretive operators which include all strongly accretive operators. In fact, it was earlier introduced by Sharma and Thakur [18] for dealing with ordinary iteration processes. In the concluding section we also include a generalization of a result of Weng [20] for dissipative mappings to approximate unique solution of $Tx = f$ and this scheme involves Mann Iteration process with errors.

Let D be a nonempty subset of a Banach space X. Recall that a mapping $T : D \to X$ is said to be strictly pseudo-contractive if there exists a constant $t > 1$ such that the inequality

$$\| x - y \| \leq \| (1 + r)(x - y) - rt(Tx - Ty) \|, \tag{1}$$

holds for all $x, y \in D$ and $r > 0$.

Let X be a Banach space with norm $\| . \|$ and dual X^*. Let $< ., . >$ denote the generalized duality pairing. For $1 < p < \infty$, the mapping $J_p : X \to 2^{X^*}$ defined by
$J_p(x) = \left\{ f^* \in X^* : Re < x, f^* > = \| f^* \| \| x \|, \| f^* \| = \| x \|^{p-1} \right\}$,
is called the duality mapping with gauge function $\phi(t) = t^{p-1}$, particularly, the duality mapping with gauge function $\phi(t) = t$, denoted by J is referred to as normalized duality mapping. In fact that $J_p(x) = \| x \|^{p-1} J(x)$ for $x \in X, x \neq 0$ and $1 < p < \infty$ (cf. [19,21,23]). A mapping T with domain D(T) and range R(T) in X is said to be accretive if for all $x, y \in D(T)$ and $r > 0$ there holds the inequality

$$\| x - y \| \leq \| x - y - r(Tx - Ty) \| . \tag{2}$$

T is accretive iff for any $x, y \in D(T)$, there is $j \in J(x - y)$ such that

$$Re < Tx - Ty,\ j > \geq \ 0. \tag{3}$$

Let D be a nonempty subset of Banach space X. Recall that a mapping $T : D \to X$ is said to be strongly accretive if there exists a real number $k > 0$ such that for every $x, y \in D$,

$$Re < Tx - Ty, j > \geq \ k \parallel x - y \parallel^2 \tag{4}$$

holds for some $j \in J(x - y)$, or equivalently, there exists a real number $k > 0$ such that for every $x, y \in D$,

$$Re < Tx - Ty, j_p > \geq \ k \parallel x - y \parallel^p \tag{5}$$

holds for some $j_p \in J_p(x - y)$. Without loss of generality, we assume that $k \in (0, 1)$. In particular, Deimling [4] proved that if X is uniformly smooth Banach space and $T : X \to X$ is strongly accretive and semicontinuous, then for each $f \in X$, the equation Tx = f has a solution in X.

Let D be a nonempty subset of a Banach Space X. A mapping $T : D \to D$ is said to be a local strongly H-accretive if for each $x \in D(T)$ and $p \in F(T)$, where F(T) is the nonempty fixed point set of T, there exists $j \in J(x - p)$ such that

$$< Tx - p, j > \geq \ k_p \parallel x - p \parallel^2 \tag{6}$$

for some $k_p > 0$, (assume $k_p \in (0, 1)$).

Now let D be a nonempty closed convex subset of a Hilbert space H, with inner product $< .,. >$ and let $T : D(T) \in H$, then T is said be locally dissipative type at a fixed point p if

$$Re < Tx - p, x - p > \leq \ C_p \parallel x - p \parallel^2 \tag{7}$$

where $C_p < 1$ and $x, p \in D(T)$. Moreover, if $\{C_n\} \subset (0, 1]$ satisfies the following conditions:

$$\lim_{n\to\infty} C_n = 0, \quad \sum_{n=0}^{\infty} C_n = \infty,$$

then the recursion

$$x_{n+1} = (1 - C_n)x_n + C_n T(x_n), \quad x_0 \in D$$

will converge to $\bar{x}$.

Dunn [5] and Rhoades and Saliga [17] further introduced the weaker version of (7),

$$Re < \xi - \bar{x}, x - \bar{x} > \leq \ C_p \parallel x - \bar{x} \parallel^2$$

for some $\bar{x} \in D(T), C_p < 1$ and for all $x \in D(T), \xi \in Tx$.

Also, Dunn [5] showed that if $x \in T(x)$ then $x = \bar{x}$. So that T can have at most one fixed point. Moreover, if $\{x_n\}$ is a sequence in D(T) satisfying

$$x_{n+1} = (1 - C_n)x_n + C_n\ \xi_n$$

where $\xi_n \in T(x_n)$, with $\{C_n\} \subset (0, 1]$ satisfying

$$\sum_{n=0}^{\infty} C_n = \infty,\ \sum_{n=0}^{\infty} C_n^2 < \infty$$

then $\{x_n\}$ strongly converges to $\bar{x}$.

In this paper we introduce the iterative solutions to the equation $Tx = f$, in the case when T is Lipschitzian and local strongly H-accretive which we shall define soon.

Let us first recall the following two iteration processes due to Mann [12] and Ishikawa [7], respectively. Here X is taken to be uniformly smooth.

(I) The Mann iteration process [12] is defined as follow: for a convex subset C of a Banach space X and a mapping $T : C \to C$, then the sequence $\{x_n\} \in C$ is defined by $x_0 \in C$,

$$x_{n+1} = (1 - C_n)x_n + C_n T_n,\ n \geq 0$$

where $\{C_n\}$ is a real sequence satisfying $c_0 = 1, 0 < c_n \leq 1$, for all $n \geq 1$ and $\sum_{n=0}^{\infty} C_n = \infty$.

(II) The Ishikawa iteration process in [21] is defined as follows:

With X and C as above, the sequence $\{x_n\} \in C$ is defined by $x_0 \in C$,

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T y_n,$$

$$y_n = (1 - \beta_n)x_n + \beta_n T x_n,\ n \geq 0,$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in (0,1] satisfying the conditions $0 \leq \alpha_n \leq \beta_n \leq 1$ for all n,

$$\lim_{n\to\infty} \beta_n = 0$$

and

$$\sum_{n=0}^{\infty} \alpha_n\ \beta_n = \infty.$$

Now we introduce the following concept of the Ishikawa iteration process with errors.

(III) The Ishikawa iteration process with errors is defined as follows:
for a nonempty subset K of a Banach space X and a mapping $T : K \to X$, the

sequence $\{x_n\}$ in K is defined by
$x_0 \in K$,
$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T y_n + u_n$,
$y_n = (1-\beta_n)x_n + \beta_n T x_n + v_n,\ n \geq 0$,
where $\{u_n\}$ and $\{v_n\}$ are two summable sequences in X. i.e.,

$$\sum_{n=0}^{\infty} \| u_n \| < \infty,\ \sum_{n=0}^{\infty} \| v_n \| < \infty$$

and $\{\alpha_n\}$ and $\{\beta_n\}$ are two sequences in [0,1] satisfying certain restrictions.

Recently, Chidume [2] proved that if $X = L_p$ (or l_p) for $p \geq 2$, then the Mann iteration process converges strongly to a solution of equation $Tx = f$ when T is Lipschitzian and strongly accretive.

1.2. Let X be an arbitrary Banach space. Recall that the modulus of smoothness $\rho_x(.)$ of X is defined by

$$\rho_x(\tau) = \frac{1}{2}\, sup \left\{ \| x+y \| + \| x-y \| -2 : x, y \in X, \| x \| = 1, \| y \| \leq \tau \right\}, \tau > 0$$

and that X is said to be uniformly smooth if $\lim_{\tau \to 0} \frac{\rho_x(\tau)}{\tau} = 0$. Recall that for a real number $p > 1$, a Banach space X is said to be p -uniformly smooth if $\rho_x(\tau) \leq d\tau^p$ for $\tau > 0$, where $d > 0$ is constant. In Xu and Roach [22] for a Hilbert space H, $\rho_H(\tau) = (1+\tau^2)^{1/2} - 1$ and hence H is 2 -uniformly smooth, while if $2 \leq p < \infty, L_p(l_p)$ is 2-uniformly smooth. In [21,22], X is uniformly smooth iff J_p is single valued and uniformly continuous on any bounded subset of X, X is uniformly convex (smooth) iff X^* is uniformly smooth (convex).

We define for positive t,

$$b(t) = sup \left\{ \frac{(\| x+ty \|^2 - \| x \|^2)}{2} - 2Re < y, J(x) > : \| x \| \leq 1, \| y \| \leq 1 \right\}.$$

Clearly $b : (0,\infty) \to [0,\infty)$ is nondecreasing, continuous and $b(ct) \leq cb(t)$, for all $c \geq 1$ and $t > 0$.

Also following Lemmas are needed to prove our results:

Lemma 1. [15] Suppose that X is a uniformly smooth Banach space and b(t) is defined as above. Then $\lim_{t \to 0+} b(t) = 0$ and

$$\| x+y \|^2 \leq \| x \|^2 + 2Re < y, J(x) > + \max\{\| x \|, 1\} \| y \| b(\| y \|)$$

for all $x, y \in X$.

Proof. The proof is same as in Reich [15], proved for a real uniformly smooth Banach space.

We also need the following Lemma for our results.

Lemma 2. [9] Let $\{a_n\}, \{b_n\}$ and $\{c_n\}$ be three nonnegative real sequences satisfying

$$a_{n+1} \leq (1 - t_n)a_n + b_n + c_n$$

with

$$\{t_n\} \subset [0, 1], \sum_{n=0}^{\infty} t_n = \infty, b_n = 0(t_n)$$

and

$$\sum_{n=0}^{\infty} c_n < \infty.$$

Then

$$\lim_{n\to\infty} a_n = 0.$$

For proof one can see Weng [20].

2. The Ishikawa iteration process with errors

In this section we study the Ishikawa iteration process with errors and prove that if X is uniformly smooth Banach space and $T : X \to X$ is a Lipschitzian local strongly H-accretive mapping, then the Ishikawa iteration process with errors converges strongly to the unique solution of the equation $Tx = f$.

Theorem 1. Let X be a uniformly smooth Banach space. Let $T : X \to X$ be a Lipschitzian local strongly H-accretive operator with a constant $k_p \in (0, 1)$ and a Lipschitz constant $L \geq 1$. Define $S : X \to X$ by $Sx = f + x - Tx$. Let $\{u_n\}, \{v_n\}$ be two summable sequences in X and let $\{\alpha_n\}, \{\beta_n\}$ be two real sequences in [0,1] satisfying:

$$(i) \lim_{n\to\infty} \alpha_n = 0, \sum_{n=0}^{\infty} \alpha_n = \infty.$$

$$(ii) \lim_{n\to\infty} sup\, \beta_n < \frac{k_p}{L^2 - k_p}.$$

For arbitrary $x_0 \in X$, the iteration sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n S y_n + u_n,$$
$$y_n = (1 - \beta_n)x_n + \beta_n S x_n + v_n, \; n \geq 0. \tag{8}$$

Moreover, suppose that the sequence $\{Sy_n\}$ is bounded, then $\{x_n\}$ converges strongly to the unique solution q of the equation $Tx = f$.

Proof. The solution of equation $Tx = f$ follows from Morales [13]. Let q denotes the solution of $Tx = f$ and the uniqueness from the local strongly H-accretiveness of T.

Now set,

$$d = sup\,\{(\| Sy_n - q \|: n \geq 0) + \| x_0 - q \|\}\,,$$

$$M = d + \sum_{n=0}^{\infty} \| u_n \| + 1. \tag{9}$$

For any $n \geq 0$, using induction, we obtain

$$\| x_n - q \| \leq d + \sum_{i=0}^{n-1} \| u_i \|,\ n \geq 0,$$

hence,

$$\| x_n - q \| \leq M,\ n \geq 0 \tag{10}$$

Now from (4), (8) and (10), we have

$Re < y_n - q, J(x_n - q) >$

$$\begin{aligned}
&= Re < x_n + \beta_n f - \beta_n Tx_n + v_n - q, J(x_n - q) > \\
&= -\beta_n Re < Tx_n - Tq, J(x_n - q) > + Re < x_n - q, J(x_n - q) > \\
&\quad + Re < v_n, J(x_n - q) > \\
&\leq -k_p\beta_n \| x_n - q \|^2 + \| x_n - q \|^2 + \| v_n \| \| x_n - q \| \\
&\leq (1 - k_p\beta_n) \| x_n - q \|^2 + M \| v_n \| \,.
\end{aligned} \tag{11}$$

Again from (4), (8) and (11), we have

$$\begin{aligned}
&Re < Sy_n - q, J(x_n - q) > \\
&= Re < Tq + y_n - Ty_n - q, J(x_n - q) > \\
&= Re < Tx_n - Ty_n, J(x_n - q) > - Re < Tx_n - Tq, J(x_n - q) > \\
&\quad + Re < y_n - q, J(x_n - q) > \\
&\leq L \| y_n - x_n \| \| x_n - q \| - k_p \| x_n - q \|^2 + (1 - k_p\beta_n) \| x_n - q \|^2 + M \| v_n \| \\
&= L \| \beta_n(Tq - Tx_n) + v_n \| \| x_n - q \| + (1 - k_p - k_p\beta_n) \| x_n - q \|^2 + M \| v_n \| \\
&= L^2\beta_n \| x_n - q \|^2 + L \| v_n \| \| x_n - q \| + (1 - k_p - \beta_n) \| x_n - q \|^2 + M \| v_n \| \\
&\leq (1 - k_p - k_p\beta_n + L^2\beta_n) \| x_n - q \|^2 + M(L + 1) \| v_n \| \,.
\end{aligned} \tag{12}$$

It then follows from (8),(9), (12) and Lemma 1 that

$$\begin{aligned}
\| x_{n+1} - q \|^2 &= \| (1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - q) + u_n \|^2 \\
&= \| (1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - q) \|^2 \\
&\quad + 2Re < u_n, J(1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - q) > \\
&\quad + \max\{\| (1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - q) \|, 1\} \| u_n \| b(\| u_n \|) \\
&\le (1-\alpha_n)^2 \| (x_n - q) \|^2 + 2\alpha_n(1-\alpha_n) Re < (Sy_n - q), J(x_n - q) > \\
&\quad + \max\{(1-\alpha_n) \| x_n - q \|, 1\} \alpha_n \| Sy_n - q \| b(\alpha_n \| Sy_n - q \|) \\
&\quad + 2 \| u_n \| \| (1-\alpha_n)(x_n - q) + \alpha_n(Sy_n - q) \| + Mb(M) \| u_n \| \\
&\le [(1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1 - k_p - k_p\beta_n + L^2\beta_n)] \| x_n - q \|^2 \\
&\quad + 2\alpha_n(1-\alpha_n)(L+1)M \| v_n \| + M^3\alpha_n b(\alpha_n) + [2M + Mb(M)] \| u_n \| \\
&\le [(1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1 - k_p - k_p\beta_n + L^2\beta_n)] \| x_n - q \|^2 \\
&\quad + M^3\alpha_n b(\alpha_n) + [LM + 2M + Mb(M)](\| u_n \| + \| v_n \|).
\end{aligned}$$

By assumption (II) on the sequence $\{\beta_n\}$, there exists $\delta \in (0, 2k)$ and a natural number $N \ge 1$ such that
$L(L^2 - k_p)\beta_n < k_p - \delta/2$, for $n \ge N$.

Consequently, we have

$$\begin{aligned}
\| x_{n+1} - q \|^2 &\le [(1-\alpha_n)^2 + 2\alpha_n(1-\alpha_n)(1-\delta/2)] \| x_n - q \|^2 \\
&\quad M^3\alpha_n b(\alpha_n) + [LM + 2M + Mb(M)](\| u_n \| + \| v_n \|) \\
&= (1 - \delta\alpha_n - \alpha_n^2 + \delta\alpha_n^2) \| x_n - q \|^2 + M^3\alpha_n b(\alpha_n) \\
&\quad + [LM + 2M + Mb(M)](\| u_n \| + \| v_n \|) \\
&\le (1 - \delta\alpha_n) \| x_n - q \|^2 + \alpha_n[M^2\delta\alpha_n + M^3 b(\alpha_n)] \\
&\quad + [LM + 2M + Mb(M)](\| u_n \| + \| v_n \|)
\end{aligned}$$

for $n \ge N$. We set $a_n = \| x_n - q \|^2, t_n = \delta\alpha_n, b_n = \alpha_n[M^2\delta\alpha_n + M^3 b(\alpha_n)]$ and $c_n = [LM + 2M + Mb(M)](\| u_n \| + \| v_n \|)$. Then the above inequality reduces to

$$a_{n+1} \le (1 - t_n)a_n + b_n + c_n, \ n \ge N.$$

Observe that $\lim_{t\to 0+} b(t) = 0$ and $\lim_{t\to\infty} a_n = 0$, so that $\lim_{t\to\infty} b(\alpha_n) = 0$. It follows from Lemma 2 that $\lim_{t\to\infty} a_n = 0$, so that $\{x_n\}$ converges strongly to the unique solution q of the equation $Tx = f$.

Corollary 1. Let X be a p - uniformly smooth Banach space with $1 < p < \infty$ and let $T : X \to X$ be a Lipschitzian local strongly H - accretive operator with a constant $k \in (0, 1)$ and a Lipschitzian constant $L \geq 1$. Define $S : X \to X$ by $Sx = f + x - Tx$. Let $\{u_n\}$, $\{v_n\}$ be two summable sequences in X and let $\{\alpha_n\}$, $\{\beta_n\}$ be two real sequences in [0,1] satisfying

$$(i)\ \lim_{n\to\infty} \alpha_n = 0,\ \sum_{n=0}^{\infty} \alpha_n = \infty$$

and

$$(ii)\ \lim_{n\to\infty}\ sup\ \beta_n < \frac{k_p}{L^2 - k_p}.$$

Then for each $x_0 \in X$, the iteration sequences $\{x_n\}$ is defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n S y_n + u_n,$$

$$y_n = (1 - \beta_n)x_n + \beta_n S x_n + v_n,\ n \geq 0, \tag{13}$$

Proof. The proof of the corollary is on the lines of Theorem 1.

3. The Mann iteration process with errors

In this section we study the Mann iteration process with errors and prove that if X is a uniformly smooth Banach space and $T : X \to X$ is a locally dissipative type mapping, then the Mann iteration process with errors converges strongly to the unique solution of the equation $Tx = f$.

Theorem 2. Let D be a uniformly smooth Banach space of X. Let $T : D(T) \to 2^D$ be a locally dissipative type operator with a constant $k \in (0, 1)$. Define $S : X \to X$ by $Sx = f + x - Tx$.

Let $\{u_n\}$ be a summable sequence in X, and $\{\alpha_n\}$ be a real sequence in [0,1] satisfying $\lim_{n\to\infty} \alpha_n = 0$, and $\sum_{n=0}^{\infty} \alpha_n = \infty$. For arbitrary $x_0 \in X$, the iteration sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n S x_n + u_n,\ n \geq 0.$$

Moreover, suppose that the sequence $\{Sy_n\}$ is bounded. Then $\{x_n\}$ converges strongly to the unique solution q of the equation $Tx = f$.

Proof. Let q be a fixed point of T. For T is a locally dissipative type mapping, we have

$$Re < Tx - q, j(x - q) > \leq C_q \parallel x - q \parallel^2 .$$

Here, $Sx = f - Tx + x$

Now,

$$
\begin{aligned}
< Sx - Sq, j(x-q) > &= Re < f - Tx + x - f + Tq - q, j(x-q) > \\
&= Re < Tq - Tx, j(x-q) > + Re < x - q, j(x-q) > \\
&\leq C_q \parallel x - q \parallel^2 + \parallel x - q \parallel^2 \\
&\leq (C_q + 1) \parallel x - q \parallel^2 \qquad (14)
\end{aligned}
$$

Now, set $d = sup\{\parallel Sx_n - q \parallel : x \geq 0\} + \parallel x_0 - q \parallel$

$$M = d + \sum_{n=0}^{\infty} \parallel u_n \parallel + 1$$

for $n \geq 0$, applying induction, we have

$$\parallel x_n - q \parallel \leq \; d + \sum_{i=1}^{n-1} \parallel u_i \parallel, \; n \geq 0$$

and hence $\parallel x_n - q \parallel \leq M, \; n \geq 0$.

Now, set

$$\beta_n = \parallel x_n - q \parallel^2 \qquad (15)$$

Because $C_n \to 0$, it is easy to show that there exists an integer $N \geq 1$ such when $n \geq N$, then

$$[1 - (1 - (C_q + 1))C_n]^2 + d^2 C_n \beta(C_n) \leq 1.$$

Let $B = \max\{\beta_i : 1 \leq i \leq N, 1\}$. First we want to show that $\beta_n \leq B^2$ and

$$\beta_{n+1} \leq [1 - (1 - (c_q + 1))C_n]^2 + B^2 d^2 C_n \beta(C_n).$$

From (8), (9), (14) and Lemma 1 for any $n \geq 0$, we have

$$
\begin{aligned}
\beta_{n+1} &= \parallel x_{n+1} - q \parallel^2 \\
&\leq \parallel (1 - C_n)(x_n - q) + C_n(Sx_n - q) + u_n \parallel^2 \\
&\leq \parallel (1 - C_n)(x_n - q) + C_n(Sx_n - q) \parallel^2 \\
&\quad + 2Re < u_n, J(1 - C_n)(x_n - q) + C_n(Sx_n - q)) > \\
&\quad + \max\{\parallel (1 - C_n)(x_n - q) + C_n(Sx_n - q) \parallel, 1\} \parallel u_n \parallel b(\parallel u_n \parallel) \\
&\leq \parallel (1 - C_n)^2 \parallel \parallel (x_n - q) \parallel^2 + 2C_n(1 - C_n) Re < Sx_n - q, J(x_n - q) > \\
&\quad + \max\{(1 - C_n) \parallel (x_n - q) \parallel, 1\} C_n \parallel (Sx_n - q) \parallel b(C_n \parallel (Sx_n - q) \parallel) \\
&\quad + 2 \parallel u_n \parallel \parallel (1 - C_n)(x_n - q) + C_n(Sx_n - q) \parallel + Mb(M) \parallel u_n \parallel
\end{aligned}
$$

$$\leq (1 - C_n)^2 \parallel (x_n - q) \parallel^2 + 2C_n(1 - C_n)(C_q + 1) \parallel (x_n - q) \parallel^2$$

$$+ \max \left\{ \parallel x_n - q \parallel^2, 1 \right\} d^2 C_n b(C_n) + 2 \parallel u_n \parallel \parallel (x_n - q) \parallel + Mb(M) \parallel u_n \parallel$$

$$\leq \left\{ (1 - C_n)^2 + 2Cn(1 - C_n)(C_q + 1) \right\} \parallel x_n - q \parallel^2$$

$$+ \max \ \left\{ \parallel x_n - q \parallel^2, 1 \right\} d^2 C_n b(C_n) + 2M \parallel u_n \parallel + Mb(M) \parallel u_n \parallel$$

$$\{1 - C_n(1 - (C_q + 1))\}^2 + \parallel x_n - q \parallel^2$$

$$+ B^2 d^2 C_n b(C_n) + \{2M + Mb(M)\} \parallel u_n \parallel,$$

for $n \geq M$, by the definition of number B, we have

$$\beta_n \leq B^2.$$

For $n \geq N$, we apply induction;
Assume $\beta_n \leq B^2$, then

$$\beta_{n+1} \leq \{1 - C_n(-(C_q + 1))\}^2 \beta_n + B^2 d^2 C_n b(C_n) + \{2M + Mb(M)\} \parallel u_n \parallel .$$

For $n > N$, we set $a_n = \beta_n, \ t_n = C_n(1 - (C_q + 1))$, $b_n = B^2 d^2 C_n b(C_n)$ and

$$C_n = \{2M + Mb(M)\} \parallel u_n \parallel .$$

Then the above inequality reduces to

$$a_{n+1} \leq (1 - t_n)^2 a_n + b_n + c_n, \ n \geq N.$$

Observe that $\lim_{n \to 0+} b(t) = 0$ and $\lim_{n \to \infty} \alpha_n = 0$. It follows from Lemma 2 that $\lim_{n \to \infty} a_n = 0$, so that $\{x_n\}$ converges strongly to the unique solution q of the equation Tx = f.

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