Isambi S. Mbalawata and Simo Sarkka (2016).

Moment Conditions for Convergence of Particle Filters with Unbounded Importance Weights. Signal Processing 118, 133--138, 2016.

In this paper, we derive moment conditions for particle filter importance weights, which ensure that the particle filter estimates of the expectations of bounded Borel functions converge in mean square and L^4 sense, and that the empirical measure of the particle filter converges weakly to the true filtering measure. The result extends the previously derived conditions by not requiring boundedness of the importance weights, but only boundedness of the second or fourth order moments. We show that the boundedness of the second order moments of the weights implies the convergence of the particle filter for bounded test functions in the mean square sense. The L^4 convergence and empirical measure convergence are assured by the boundedness of the fourth order moments of the weights. We also present an example class of models and importance distributions where the moment conditions hold, but the boundedness does not. The unboundedness in these models is caused by point-singularities in the weights which still leave the weight moments bounded. We show by using simulated data that the particle filter for this kind of model also performs well in practice.

Keywords: Particle filter; Convergence; Unbounded importance weights; Moment condition