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Modelling Stock Returns Volatility on Uganda Securities Exchange

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Abstract

Stock returns volatility of daily closing prices of the Uganda Securities Exchange(USE) all share index over a period of 04/01/2005 to 18/12/2013 is Modelled. We employ different univariate Generalised Autoregressive Conditional Heteroscedastic(GARCH) models; both symmetric and asymmetric. The models include; GARCH(1,1), GARCH-M, EGARCH(1,1) and TGARCH(1,1). Quasi Maximum Likelihood(QML) method was used to estimate the models and then the best performing model obtained using two model selection criteria; Akaike Information criterion(AIC) and Bayesian Information criterion(BIC). Overall, the GARCH$(1,1)$ model outperformed the other competing models. This result is analogous with other studies, that GARCH$(1,1)$ is best.

Keywords: Modelling, Volatility, Uganda Securities Exchange

1 Introduction

The financial sector all over the world is working tirelessly to see to it that success is recorded in any financial activity engaged in. To this regard, one of the important aspects that need special attention by any investor or policy maker is the volatility of stock returns. Volatility can be defined as a statistical measure of the dispersion of returns for a given security or market index and it can either be measured using the standard deviation or variance between returns from that same security or market index, [1].

[2], assert that modelling volatility in financial markets is important because it sheds further light on the data generating process of the returns. Volatility forecasting is an important area of research to financial markets and a lot of effort has been expended in improving volatility models since better forecasts translates into better pricing of options and better risk management.

Volatility has been used as a proxy for riskiness associated with the asset, [3]. The conditional variance of financial time series is important for pricing of derivatives, calculating measures of risk, and hedging. It is for this reason that enormous volatility models have been developed by many renown researchers and scholars since Engle's seminal paper of 1982.

ARCH model proposed by [4] and its extension; GARCH model by [5], were the first models to be introduced into the literature. These were found to be useful and have become very popular in that they enable analysts to estimate the variance of a series at a particular point in time, [6]. Since then, there have been a great number of empirical applications of modelling and forecasting the volatility of financial time series by employing these models and their many specifications, [1].

[7], state that many stock markets over the last two decades have been established in Africa due to financial sector development and reforms in many Sub-Saharan Africa (SSA) countries aimed at shifting their financial systems from one of bank-based to security market-based. Most of the emerging stock markets in Africa are at the early stages of taking off striving to register success. Knowledge on volatility of stock returns is a crucial area of concern that needs special attention to compete favorably with developed stock markets.

The main objective of this paper is to model stock returns volatility on the USE using different specifications of the univariate GARCH models both symmetric and asymmetric.

The rest of the paper is organized as follows: Section 2 presents the methodology of the current study, Section 3 presents the data analysis while Section 4 concludes the study.

2 Methodology

2.1 Symmetric GARCH Models

For the symmetric models, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset. Under this study, there are two symmetric GARCH models that were used; the standard GARCH$(1,1)$ and GARCH-M$(1,1)$. The models are discussed below:

2.1.1 GARCH $(1,1)$ Model

The basic and most widespread model GARCH $(1,1)$ is given by;

$$\text{Mean} \quad \text{equation} \qquad r_t = \mu + \varepsilon_t \tag{1}$$

$$\text{Variance} \quad \text{equation} \qquad \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{2}$$

where $\omega > 0, \alpha_1 \geq 0$ and $\beta_1 \geq 0$ and r_t is the return of the asset at time t, μ is the average return. σ_t^2 is the conditional variance, $\varepsilon_t = \sigma_t Z_t$ are the residual returns. For GARCH$(1,1)$, the constraints $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ are needed to ensure that conditional variance is positive, [8].

2.1.2 GARCH$-$in$-$Mean (GARCH$-$M$(1,1)$) Model

The return of a security may depend on its volatility, [9]. Thus, one may consider the GARCH$-$M Model of [10] in order to model such a phenomenon, where "M" stands for GARCH in the mean, [11]. It is an extension of the basic GARCH model and allows the conditional mean of a sequence to depend on its conditional variance or standard deviation. The simple GARCH$-$M$(1,1)$ model is given by;

$$\text{Mean} \quad \text{equation} \qquad r_t = \mu + \xi \sigma_t^2 + \varepsilon_t \tag{3}$$

The variance equation is similar to that defined for the GARCH$(1,1)$ process in Equation (2). The parameter ξ is called the risk premium parameter. A positive ξ indicates that the return is positively related to its volatility, that is; a rise in mean return is caused by an increase in conditional variance as a proxy of increased risk, [1]. [10], assume that the risk premium is an increasing function of the conditional variance of ε_t. [6], also contends that, the greater the conditional variance of returns, the greater the compensation necessary to induce the agent to hold the long term asset.

2.2 Asymmetric GARCH models

One of the shortcomings of symmetric GARCH models is that they are unable to capture the asymmetry or leverage effects and yet such effects are believed to be very important in studying the behavior of stock returns. This has led to the introduction of a number of models; asymmetric models, to deal with this phenomenon. For asymmetric models, the shocks of the same magnitude, positive or negative, have different effects on future volatility. Under this study, EGARCH by [12] and TGARCH by [13] and [14] models are employed to capture the asymmetric phenomenon in the USE returns.

2.2.1 The Exponential GARCH (EGARCH(1,1)) Model

This model captures the asymmetric responses of the time varying variance and at the same time, it ensures that the conditional variance is always positive even if the parameter values are negative, [15]. This means that there is no need for parameter restrictions to impose non negativity. It was developed by [12] with the following specification:

$$\ln\left(\sigma_t^2\right) = \omega + \beta_1 \ln\left(\sigma_{t-1}^2\right) + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right\} - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}, \tag{4}$$

where $\pi = \dfrac{22}{7}$, γ is the asymmetric response parameter or leverage parameter. In most empirical cases, γ is expected to be positive so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty, [1].

2.2.2 The Threshold GARCH (TGARCH(1,1)) Model

This was developed by [13] and [14]. It a special case of APARCH by [16] and below is its model specification

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{5}$$

where d_{t-1} is a dummy variable, *i.e.*,

$$d_{t-1} = \begin{cases} 1 & \text{if} \quad \varepsilon_{t-1} < 0, \quad \text{bad news} \\ 0 & \text{if} \quad \varepsilon_{t-1} \geq 0, \quad \text{good news.} \end{cases}$$

Again γ is the asymmetric response parameter or leverage parameter. The model reduces to the standard GARCH form when $\gamma = 0$. Otherwise, when the shock is positive (*i.e.*, good news) the effect on volatility is α_1, but when the shock is negative (*i.e.*, bad news) the effect on volatility is $\alpha_1 + \gamma$. [17], asserts that when γ is significant and positive, negative shocks have a larger effect on σ_t^2 than positive shocks.

3 Data Analysis and Results

3.1 Data

Secondary data from the USE was used. Daily closing prices of USE All share index data over a period of 9 years extending from 04/01/2005 to 18/12/2013(1426 observations) was used. The USE All Share Index is the major stock market index on the USE. It tracks the daily performance of the 16 most capitalized companies listed on the USE.

Asset returns

Most financial studies involve returns, instead of prices, of assets. This is because the return of an asset is a complete and scale-free summary of the investment opportunity for average investors and return series are easier to handle than price series because the former have more attractive statistical properties, [18]. Let P_t and P_{t-1} denote the closing market index of USE at the current (t) and previous day $(t-1)$, respectively. The USE All Share returns (log returns or continuously compounded returns) at any time are given by:

$$r_t = log\Big(\frac{P_t}{P_{t-1}}\Big) \tag{6}$$

3.2 Basic statistics of USE returns series

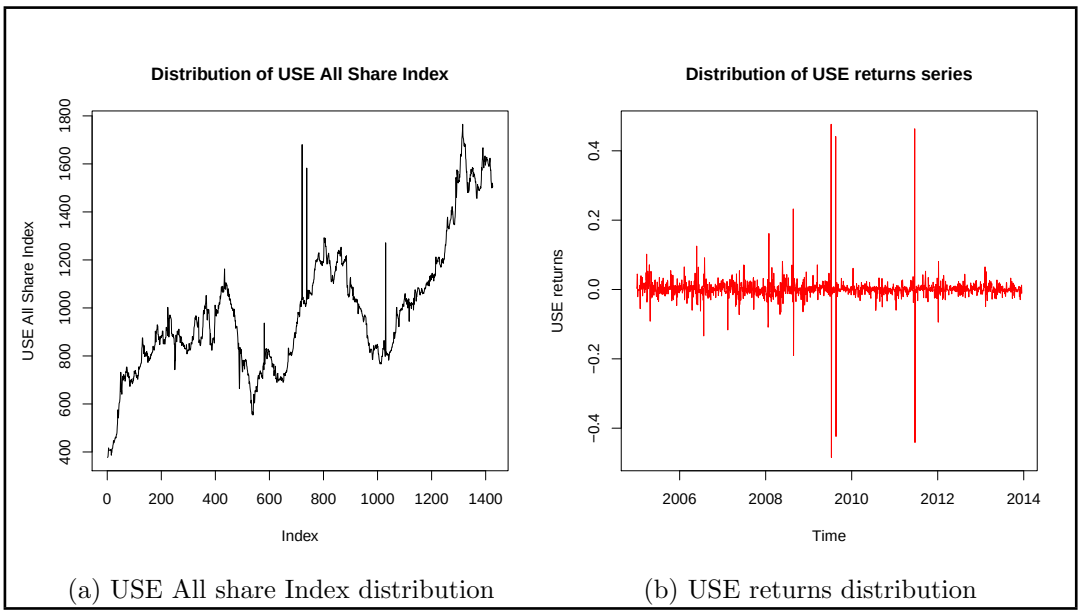
Descriptive statistics for the returns was carried out to describe the behavior of USE return series. The skewness, kurtosis and Jarque-Berra test for normality were used as the diagnostic tools under this study.

Table 1: Descriptive statistics of USE returns Series

Mean	0.00 09705
median	0.0002228
Maximum	0.4766000
minimum	-0.4844000
Std.Dev	0.03649952
Skewness	0.3190972
Kurtosis	102.9358
Jarque-Bera	630969.7487
JB probability	<2.2e-16
No. of observations	1425

Sample: Jan 04, 2005 to Dec 18, 2013

Descriptive statistics for the USE return series are shown in Table (1). As is expected for a time series of returns, the mean is close to zero. The return series are positively skewed an indication thet the USE has non-symmetric returns. The kurtosis is greater than three for the normal distribution. This indicates that the underlying distribution of the returns are leptokurtic or heavy tailed. The series is non-normal according to the JB test which rejects normality at the 1% level for the series.

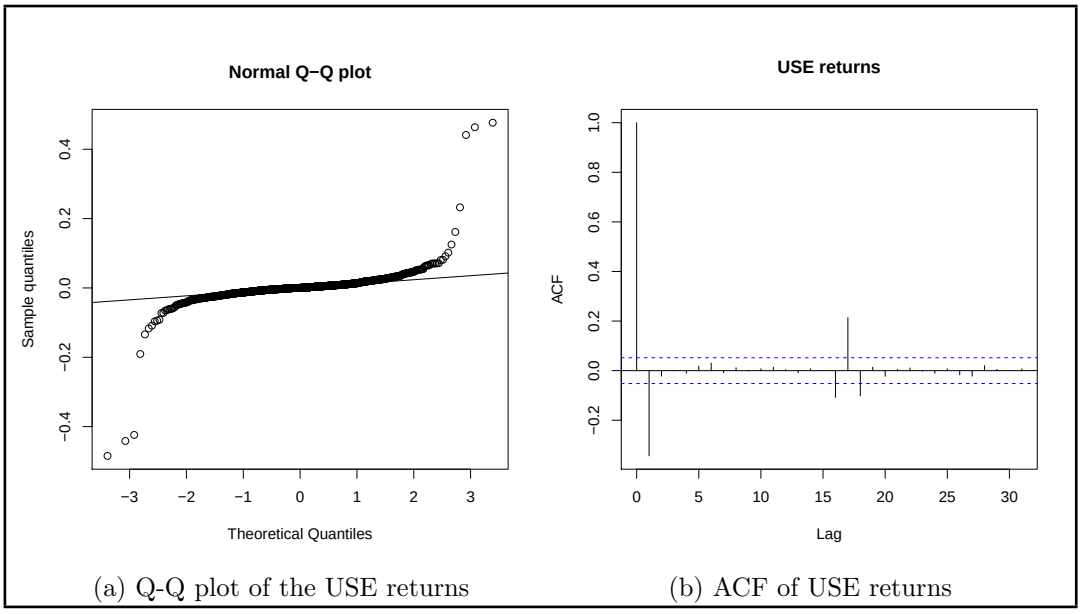


(a) USE All share Index distribution (b) USE returns distribution

Figure 1: USE Daily prices and returns distributions(Jan.2005-Dec.2013)

Figure (1) shows the distribution of the USE All share index Figure (1a) and the return series 1b. From the graphs, the stock prices are non stationary while the return series are stationary with a mean return of zero. There is also evidenced volatility clustering in the return series. This is analogous to other studied stock exchanges.

The Q-Q graphical examination was employed to further check whether the USE index return series is normally distributed. According to [19], a Q-Q plot is a scatter plot of the empirical quantiles against the theoretical quantiles of a given distribution. [1], assert that if the sample observations follow approximately a normal distribution with mean equal to the empirical mean and standard deviation equal to the empirical standard deviation, then the resulting plot should be roughly scattered around the 45-degree line with a positive slope, the greater the departure from this line, the greater the evidence for the conclusion that the series is not normally distributed. The behaviour of the USE return series can also be deduced using a correllogram. This helps in establishing whether there is serial correlation in the series.



(a) Q-Q plot of the USE returns (b) ACF of USE returns

Figure 2: Q-Q and ACF plots of USE(Jan.2005-Dec.2013)

The Q-Q plot in Figure (2a) shows that the return distribution of USE exhibit fat tails confirming the results in Table(1) that the USE returns data do not follow the normal distribution. From Figure (2b) , there is little evidence of serial correlation in the USE return series except at lags 1, 16, 17 and 18.

3.3 Testing for stationarity

Before estimating the parameters of the models under consideration, it is required that one checks whether the series are stationary. Under this study, ADF test, [20] was used to investigate whether the daily price index and its returns are stationary series. The ADF test includes a constant term without trend.

Table 2: ADF unit root test for the USE All Share index and returns series

		Critical values		
Series	ADF Statistic	1%	5%	10%
Price Index	-2.2484(11)	-3.43	-2.86	-2.57
Return series	-10.1458(11)	-3.43	-2.86	-2.57

Note: Critical values are taken from [21]

Table 2 shows the ADF test results for both the USE All share index and the return series. The ADF test for the price index indicate that they have to be considered as non-stationary. On the other hand, the null hypothesis

of a unit root is rejected for the return series at all levels of significance. This means that the return series might be considered as stationary over the specified period.

3.4 Testing for Heteroscedasticity

It is always sensible to pre-test the data if ARCH effects are suspected in a series. [4], proposes a Lagrange Multiplier (LM) test for ARCH. Therefore the LM test was used to test for ARCH effects. Below is the procedure for the LM test: Let $\varepsilon_t = r_t - \mu$ be the residuals of the mean equation. The squared series ε_t^2 is then used to check for conditional heteroscedasticity, which is also known as the ARCH effects. This test is equivalent to the usual F statistic for testing $\alpha_i = 0 (i = 1, \ldots q)$ in the linear regression

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + e_t, \qquad t = q+1, \ldots, T \tag{7}$$

e_t is the error term, q is the pre-specified integer and T is the sample size. The null hypothesis that there are no ARCH effects up to order q can be formulated as: $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_q = 0$ against the alternative $H_1 : \alpha_i > 0$, for at least one, $i = 1, 2, \ldots, q$. The test statistic for the joint significance of the q-lagged squared residuals is given by TR^2 where T is the number of observations and is evaluated against the $\chi^2(q)$ distribution.
In order to test for ARCH effects, ARMA(1,1) model for the conditional mean in the return series was employed as an initial regression. Then the null hypothesis that there are no ARCH effects in the residual series up to lag 12 was tested. The results are summarised in Table 3. The null hypothesis is rejected

Table 3: ARCH-LM Test for residuals of the USE returns Series

ARCH-LM test statistic	Prob.chi-square(12)
128.9103	$<$ 2.2e-16
H_0 : There are no ARCH effects	

basing on the ARCH-LM test results in Table 3. This means that there are ARCH effects in the residual series of the mean equation, an indication that the variance of the USE return series is non-constant.

3.5 Model estimation

The parameter estimates of the different models for the USE returns for the study period are showed in the table below. The diagnostics test results of the models, including the AIC and BIC are also provided.

Table 4: Estimation results of different GARCH Models for USE

Models	GARCH	GARCH-M	EGARCH	TGARCH
		Mean Equation		
μ	-0.0005361	0.01402*	-0.000239	0.001001
λ	-	0.2438**	-	-
		Variance equation		
ω	7.725e-06	1e-06	-0.1404^{*}	$7.0e-05^{*}$
α	0.6135***	0.1882*	0.02353	0.2420*
β	0.7629*	0.7643*	0.96*	0.8002*
γ	-	-	0.510324*	-0.08638
$\alpha+\beta$	1.3764	0.9525	0.9835	1.0422
		ARCH-LM test		
Statistic	0.4075927	0.14297	0.11248	0.1535
Probability	0.9999999	0.9996	0.9998	0.9995
		Model Performance		
LL	3094.705^{1}	2396.472^{4}	3087.225^{2}	3053.652^{3}
AIC	-4.338^{1}	-3.3565^{4}	-4.326^{2}	-4.2788^{3}
BIC	-4.323^{1}	-3.3380^{4}	-4.308^{2}	-4.2603^{3}

Note: * denotes significance at 1% level, ** at 5% level and *** at 10% level, superscripts denote rank of the model

From Table 4 , the mean is insignificantly different from zero for all the models under study except for the GARCH-M model. Except for ω, the coefficients α and β in the variance equation of the GARCH$(1,1)$ are statistically significant at the 10% and 1% levels respectively. It is also noted that all the coefficients have the expected sign. The significance of α and β indicates that news about volatility from the previous period have an impact on the current volatility. Also the persistence, $\alpha+\beta>1$ meaning that the volatility process is explosive suggestive of an integrated process.

In the GARCH-M$(1,1)$ model, the risk premium, λ is significant at the 1% level and positive, indicating that volatility used as a proxy for risk of returns is positively related to the returns. These results are consistent with the asset price theorem, which states that the returns of an asset depend on the level of risk as the asset takes.

The asymmetric EGARCH$(1,1)$ model estimated for the USE returns indicates that all the estimated coefficients ω and β are statistically significant at the 1% confidence level. However, α is nonsignificant. The leverage parameter, γ is also statistically significant with a positive sign, suggesting the presence of leverage effects in the

The TGARCH$(1,1)$ model; an alternative asymmetric model estimated also indicates that all the coefficients and are significant at 1% and have the

expected signs. The persistence is very long and explosive suggestive of an integrated process. The leverage parameter is negative and insignificant,suggesting absence of leverage effects.

The ARCH-LM statistic for all the GARCH models under study also reported in Table 4 indicate that there are no additional ARCH effects remaining in the residuals of the the models meaning that the models were well specified.

Overall, using the maximum LL, minimum AIC and BIC as the model selection criteria, GARCH$(1,1)$ outperformed the other models in modelling volatility of USE returns for the study period.

4 Conclusion

The volatility of USE returns have been modelled for a period of 04/01/2005 to 18/12/2013 using both symmetric and asymmetric univariate GARCH models that capture the most common stylised characteristics of index returns. The models are GARCH$(1,1)$, GARCH-M$(1,1)$, EGARCH$(1,1)$ and TGARCH$(1,1)$.

Basing on the the empirical results obtained, the following can be concluded: Firstly, it was found that the USE returns are non-normal and heteroscedasticity was found to be present in the residual return series and therefore the support for the use of the above models. Secondly, the USE return series also exhibit volatility clustering and leptokurtosis as evidenced from the high kurtosis values. Thirdly, the parameter estimates for the GARCH$(1,1)$ model $\alpha + \beta > 1$ indicating that the volatility of USE stock returns is an explosive process. Forth, the risk premium parameter, λ in the GARCH-M$(1,1)$ model is statistically significant and positive implying that an increase in volatility is affected by an increase in returns. This result is in agreement with the asset price theory. Fifth, there is evidence of leverage effects based on the estimation results of the leverage parameter γ of the EGARCH$(1,1)$ model except for the TGARCH$(1,1)$ model. The GARCH$(1,1)$ model outperformed the other competing models in modelling volatility of USE returns. This result is analogous with other studies, that GARCH$(1,1)$ is best.

It is recommended that IGARCH$(1,1)$ is used in place of GARCH$(1,1)$ to adequately describe the USE returns. Further studies can also be done on forecasting the volatility of the USE returns.

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