A FREQUENCY DOMAIN TECHNIQUE FOR STRUCTURAL PARAMETER UPDATING

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ABSTRACT

A technique is described to update the mass and stiffness parameters of a dynamic model using *frequency response functions*. Previous investigators have presented frequency domain techniques on the basis of finding the parameters which minimize either the errors in the equation of motion, equation error algorithms, or the errors in the frequency domain response, output error algorithms. Since, in practice, the theoretical degrees-of-freedom are not compatible with the measured degrees of freedom, it is necessary, with the equation error algorithms, to perform model reduction or expanding the measurement vectors. Furthermore, measurement noise result in non independent equation errors and hence biased estimates. The output error algorithm, however, require enormous computational effort and is known to have convergence difficulties. The technique *presented in this paper use the receptance* data and overcome the limitations of the output error and the equation error algorithms. The technique is an improvement on the new approach described by the author recently. Both error free and noisy data are used in the simulation examples.

NOMENCLATURE

[ ]	matrix
{ }	column vector
( )	single element (of a matrix)
[M]; [K]	mass; stiffness matrices
[ΔK]	perturbing stiffness matrix
[S]	sensitivity matrix
[W]	weighting matrix
$\{F_j\}_1$	*forcing vector with zero elements* except unity at coordinate j
{Y}	observation vector
{z}	vector of unknown mass and stiffness parameters
{Δz}	$\{z\} - \{z_A\}$
EI	flexural rigidity
M	mass per unit length
ω	vibration frequency (in $rad.s^{-1}$)
α	receptance
Δ	difference (of matrices, vectors, scalars)
N	number of degrees of freedom
g	number of unknown mass and stiffness parameters
P	product of remainders
ε	error
$\frac{\partial}{\partial z}$	partial differentiation w.r.t to z
[K]'	$\frac{\partial [K]}{\partial z}$
[M]'	$\frac{\partial [M]}{\partial z}$
[ΔK]'	$\frac{\partial [\Delta K]}{\partial z}$
$\sum_{j=1}^{g}$	summation from j = 1 to g

subscript

j, k, l	integers
jj	element in column j and row j of a matrix
X	tested structure, experimentally derived
A	current theoretical model
I	initial theoretical model
U	updated model
α	reference to receptance
Y	reference to elements in {Y}
B1 - B5	reference to beam elem. no. 1 - no. 5

superscript

T	matrix transpose
-1	matrix inverse
'	matrix differentiation

Abbreviations

DOF(s)	degree(s) of freedom
MAC	modal assurance criterion
FRF	frequency response function

1. INTRODUCTION

The use of the frequency response function, as opposed to the modal data, to update structural model parameters is increasingly receiving considerable interest. One advantage of using the FRF instead of the modal data is the availability of a large data sample and the avoidance of modal extraction routine and its associated errors. The FRF may be measured more accurately than the mode shape vectors extracted from the FRF.

An algorithm which use the FRF to update the parameter has to address, among others, two problems. First the large mismatch between the number of theoretical and measured degrees-of-freedom and secondly the experimental errors in the measured data. Most of the frequency domain techniques [1-7] are based on finding the parameters which minimize either the errors in the equation of motion (equation errors) or the errors in the response vectors between the updated model and the real structure (output errors).

Since there is a large mismatch between the number of theoretical and measured DOFs, the full order matrix equation of motion cannot be formulated using measured response data. This problem has been dealt with by either condensing the equation to the measured DOFs, for example using the theoretical modal matrix in the transformation of coordinates [1-3], or expanding the response vector by estimating the response data of the unmeasured DOFs. In either case the resulting equations are linear in the unknown parameters, with the coefficients derived using the experimental data which is contaminated with measurement noise. The equation errors, in this case, are non independent and contribute to biased estimates.

While the output errors can be formulated for selected coordinates which correspond to the measured DOFs, the equations are nonlinear in the parameters and the solution is effected iteratively using, for example, a truncated Taylors expansion [3-5]. The solution process is time consuming and involves a large computational effort as at each discrete frequency, inversion of a matrix of order N has to be performed. The output error algorithm, in most cases, suffer from parameter divergent problem as the assumptions in the linearization are rarely satisfied.

This paper present an alternative approach. The forced equation of motion is rewritten as a homogeneous equation of motion of a structure with added stiffness at the excitation coordinate. The eigenvector of the structure with added stiffness is therefore given by the receptance vector of the structure without the added stiffness, and the eigenvalue is given by the square of circular frequency corresponding to the receptance data. It will be seen that the added stiffness is simply given by the reciprocal of the receptance at the excitation coordinate and bearing an opposite sign. We then imagine to add a similar stiffness to the theoretical model. We then perform eigenvalue sensitivity analysis to adjust the theoretical parameters for the system with added stiffness. However, no eigendata is actually measured and the theoretical eigendata for sensitivity analysis is readily available from the FRF. This approach result in a numerically stable algorithm while avoiding the limitations of the equation error formulation.

The present approach is derived from the author's experience with the modal based mass and stiffness addition technique [8,9]. The modal based approach, however, is computationally demanding as an eigensolution has to be found for each added stiffness. The idea of a frequency domain approach from the stiffness addition concept was previously presented [10]. The present paper describes an improved algorithm and also takes into consideration randomness in the FRF data due to measurement noise.

2. THEORY

The equation of motion of a system excited at a single coordinate j is given, using receptances, by:

$$[\,[K] - \omega^2\,[M]\,]\{\alpha(\omega)\} = \{F_j\}_1 \qquad (1)$$

Equation (1) can be re-arranged as:

$$[\,[K] + [\Delta K] - \omega^2\,[M]\,]\{\alpha(\omega)\} = \{0\} \qquad (2)$$

where $$[\,\Delta K\,]\{\alpha(\omega)\} = -\{F_j\}_1 \qquad (3)$$

Thus, at any frequency ω, the forced equation of motion can be transformed into a homogeneous equation of motion of the system perturbed by a stiffness matrix $[\,\Delta K\,]$.

However, from (3), there is an infinite set of perturbing stiffness matrices which satisfy the equation. We select the simplest case of perturbation, by a lumped spring grounded at a single coordinate. From (3), this coordinate must be the excitation coordinate, j, and the added stiffness is:

$$\Delta K_{jj} = -\frac{1}{\alpha_{jj}(\omega)} \qquad (4)$$

If we consider now the FRF measured on a real structure, any discrete frequency, ω_x, and its corresponding receptance vector can therefore be treated as a natural frequency and mode shape respectively, of the structure grounded at the excitation coordinate, by a stiffness defined by (4). We add a similar stiffness to the theoretical model, $(\Delta K_{jj})_A$, and locate a corresponding natural frequency of the perturbed theoretical model. The frequency and point receptance of the theoretical model must satisfy (5). The frequencies are used to update the parameters by eigenvalue sensitivity analysis.

$$(\Delta K_{jj})_X = (\Delta K_{jj})_A = -\frac{1}{(\alpha_j(\omega))_A} \qquad (5)$$

However, as the FRF data is evaluated at discrete intervals, it is unlikely that for each stiffness $(\Delta K_{jj})_X$ from the measured FRF, a frequency corresponding to an identical stiffness in the theoretical FRF data will be located. Thus, we locate a frequency ω_A giving $(\Delta K_{jj})_A$ which is very close but not necessarily identical to $(\Delta K_{jj})_X$. This frequency must also be the one belonging to a correlated *mode shape*. The mode shapes to be correlated are the receptance vectors of the theoretical model, evaluated at frequency ω_A, and measured receptance at a corresponding frequency ω_X. Therefore, in general, $[\Delta K_X]$ from the measured structure is not identical but close to $[\Delta K_A]$ from the theoretical model. The eigenvalue sensitivity analysis must therefore be performed with respect to the parameters of the structure as well as the parameters of the perturbing matrix, and is given by:

$$\frac{\partial(\omega^2)}{\partial z} = \frac{\{\alpha_A\}^T[\,[K]' + [\Delta K]' - \omega^2\,[M]'\,]\{\alpha_A\}}{\{\alpha_A\}^T[M_A]\,\{\alpha_A\}} \qquad (6)$$

where the denominator represent mass normalization of the *mode shape* vector. Therefore, at measurement frequency ω_X sensitivity analysis result in;

$$\Delta\omega^2 = \{\alpha_A\}^T\left[\sum_{j=1}^{g}\frac{\partial[K]}{\partial z}\,\Delta z_j + \frac{\partial[\Delta K]}{\partial(\Delta K_{jj})}\Delta(\Delta K_{jj})\right.$$

$$\left. - \omega_A^2\sum_{j=1}^{g}\frac{\partial[M]}{\partial z}\,\Delta z_j\right]\{\alpha_A\}/\{\alpha_A\}^T[M_A]\{\alpha_A\} \qquad (7)$$

The fact that [M], [K] and [ΔK] are all close to their theoretical counterpart means that the error in first order approximations in (7) is very small and therefore we can comfortably use the equation to adjust the parameters iteratively.

But $\{\alpha_A\}^T\ [\frac{\partial[\Delta K]}{\partial(\Delta K_{jj})}\Delta(\Delta K_{jj})]\ \{\alpha_A\}$

$$= (\alpha^2_{jj})_A((\Delta K_{jj})_X - (\Delta K_{jj})_A) \qquad (8)$$

Therefore (7) simplify to:

$$\frac{\{\alpha_A\}^T\left[\sum_{j=1}^{g}\frac{\partial[K]}{\partial z}\,\Delta z_j - \omega_A^2\sum_{j=1}^{g}\frac{\partial[M]}{\partial z}\,\Delta z_j\right]\{\alpha_A\}}{\{\alpha_A\}^T[M_A]\{\alpha_A\}}$$

$$= \omega_X^2 - \omega_A^2 + \left(\frac{(\alpha_{jj})_X - (\alpha_{jj})_A}{\{\alpha_A\}^T[M_A]\{\alpha_A\}}\right)\frac{(\alpha_{jj})_A}{(\alpha_{jj})_X} \qquad (9)$$

Equation (9) is applied to each discrete frequency in the measurement range and result in an overdetermined matrix equation, (10), from which a vector of parameter update, $\{\Delta z\}$, for the current theoretical model can be determined.

$$[S]\{\Delta z\} = \{Y\} \qquad (10)$$

As previously stated, the frequencies ω_X and ω_A must be from a pair of correlated *modes shapes* (FRF vectors).

3. CORRELATING THE FRF VECTORS

Correlating the FRF in order to identify ω_A which relates to ω_X is necessary because the sensitivity algorithm is essentially modal based. However, mode shapes of the perturbed structure and the perturbed theoretical model are readily given by the receptance vectors. Thus, the correlation is performed using the receptances using the modal assurance criterion (MAC). The correlation is performed for each discrete frequency ω_X. The MAC in terms of the receptance vectors as mode shapes become;

$$\text{MAC} = \frac{|\{\alpha_A\}^T.\{\alpha_X\}\,|^2}{|\{\alpha_A\}^T\{\alpha_A\}\,|.|\{\alpha_X\}^T\{\alpha_X\}|} \qquad (11)$$

The process of determining the correlated FRF is as follow. A set of FRF is measured on a structure at the excitation coordinate and a number of other control coordinates. The theoretical FRF is also computed at the excitation coordinate and the other coordinates which match measured coordinates. Starting at the lower limit of the measured FRF, theoretical receptance at the excitation coordinate is compared with measured receptance to find a frequency at which (5) is best satisfied. That is:

For each l, starting at the lower limit of the range as $l = 1$, we scan the theoretical receptance and compute a product P_l for each k, where $k = 1$ is the lower limit in the theoretical FRF.

$$P_l = ((\alpha_{jj}(\omega_l))_X - (\alpha_{jj}(\omega_k))_A)$$

$$* ((\alpha_{jj}(\omega_l))_X - (\alpha_{jj}(\omega_{k+1}))_A)$$

where $k = 1, 2, \ldots$ etc.

If P_l is negative, then (5) is satisfied in the neighborhood of ω_k and ω_{k+1}. In this case we

chose either ω_k or ω_{k+1} as the theoretical natural frequency, ω_A, which is to be correlated to the experimental frequency $\omega_X = \omega_l$, depending on whether $(\alpha_{jj}(\omega_k))_A$ or $(\alpha_{jj}(\omega_{k+1}))_A$ is closer to the measured receptance $(\alpha_{jj}(\omega_l))_X$. This choice, however, has to be confirmed by $(MAC)_l$.

If $(MAC)_l < 0.9$	ignore and proceed with next k.
If $(MAC)_l > 0.9$	a correlated FRF vector has been determined. Proceed to next l, reset k.

However, once a correlated theoretical data is determined, it is not necessary to always reset $k = 1$ to the lower limit of the theoretical FRF. This will make the process time consuming. It has been found that considerable time is saved if subsequent scanning of the theoretical FRF starts just a few hertz before the current ω_A. The process has been successfully automated.

4. CONSIDERATION OF MEASUREMENT NOISE

As opposed to the equation error algorithm, measurement noise only appear on the right-hand-side of (9). If measured quantity, $(\alpha_{jj})_X$, *changes by* ε_α *the right-hand-side of* (9) will change by ε_Y, where;

$$\varepsilon_Y = \frac{-1}{\{\alpha_A\}^T[M_A]\{\alpha_A\}} \frac{(\alpha_{jj}^2)_A}{(\alpha_{jj}^2)_X} \varepsilon_\alpha \qquad (12)$$

Therefore, if ε_α *is a random measurement noise* with a mean of zero, the expected value of ε_Y becomes zero. Hence the least squares solution of (10) is an unbiased estimate. Practical solution, however, involve the consideration of the theoretical parameters in the updating process, using constrained optimization in a Bayesian framework. Equation (12) can also be used to estimate the standard deviation of the observation vector $\{Y\}$ if ε_α is replaced by the standard deviation of measured receptance at the excitation coordinate. Let W_Y and W_1 be diagonal weighting matrices of the observation vector $\{Y\}$ *and the initial theoretical parameters* respectively. Parameter update for the current theoretical model, incorporating a constraint of minimum changes from the initial estimates is:

$$\{\Delta z\} = [[S]^T[W_Y][S] + [W_1]]^{-1}\{[S]^T[W_Y]^T\{Y\} + [W_1]^T\{z_A - z_1\}\} \qquad (13)$$

5. SIMULATION EXAMPLE

A plane frame made of five beam members, B1 to B5, was modeled by a 39 DOF FE model using 14 beam elements, where only motion in the plane of the *frame was considered, figure* 1. The frame was modeled to simulate, as much as possible, a free test condition. However, to simplify the computation of the FRF, rigid body modes with zero natural frequencies were avoided by adding soft springs at DOFs 4, 10 and 39. Parameters used to simulate the frame and the initial theoretical model are given in table 1. Two *excitation coordinates, 24 and 31, were chosen* (only one coordinate excited at a time).

Two updating cases were performed. Case 1 assumed error-free data and case 2 considered 10 percent random errors with zero mean in the simulated receptances.

5.1. UPDATING CASE 1 (Error free data)

Simulated point receptances at coordinates 24 and 31 were used to update the initial mass and flexural parameters of the five beam members. The receptances were simulated to cover the *frequency range of 50 Hz to 150 Hz at 0.5 Hz* interval, but the data used to define ω_X was taken at 2 Hz interval. To obtain correlated theoretical data, additional receptances were simulated to have been measured at coordinates 1, 4, 7, 10, 13, 18, 25, 28, 34, 37 and these together with the point receptances and the *theoretical receptances were used to compute* MAC. The theoretical parameters were updated by unconstrained optimization, using (10), and converged to the exact simulated parameters.

5.2. UPDATING CASE 2 (With 10 % random errors)

Random errors with zero mean and 10 percent standard deviation were added to the simulated receptances, both at the excitation coordinates and at the other coordinates used to compute the MAC. Figure 2 compares simulated and theoretical receptances at each excitation coordinate. The frequencies were correlated by MAC using the contaminated simulated data. The parameters were updated using a constraint of minimum changes from the initial estimates using (13). The weighting matrix for the observation vector $\{Y\}$, was determined from the knowledge of the standard deviations of the elements of $\{Y\}$. The standard deviations of the elements of $\{Y\}$ were obtained by replacing ε_α in (12) by 10 percent of each receptance data (at excitation coordinates) as standard deviation and computing ε_Y as standard deviations of $\{Y\}$. The weighting matrix W_1 was established by assigning confidence limits, as standard deviations, on the initial parameters (shown in brackets in table 1).

The parameters updated in 7 iterations are sufficiently accurate and are shown in table 2. Figure 3 shows the convergence of the parameters. Figure 4 compare rotational receptance prediction between the initial model, updated *model and the simulated system* at coordinate 11 for an excitation at coordinate 31.

6. CONCLUSION

A frequency domain technique for structural parameter updating has been presented. The technique treats receptance vectors as eigenvectors of a system perturbed by grounded stiffness at the excitation coordinate. The parameters are adjusted by employing eigenvalue sensitivity analysis on the perturbed system but no eigenvalue is actually measured. The approach presented has several advantages over the traditional modal based techniques. First it enables maximum utilization of the data over the whole measurement range than is the case with techniques which use modal data of the unperturbed system alone. Secondly, modal identification errors are avoided as the technique use receptances. Comparing to other frequency domain methods, the present technique does not require full length measurement vectors, does not require model reduction and avoids correlated equation errors and the problem of bias. The technique has been tested using data simulated with measurement errors with very good results. Future work is to extend the algorithm to incorporate damping and test on a real structure.

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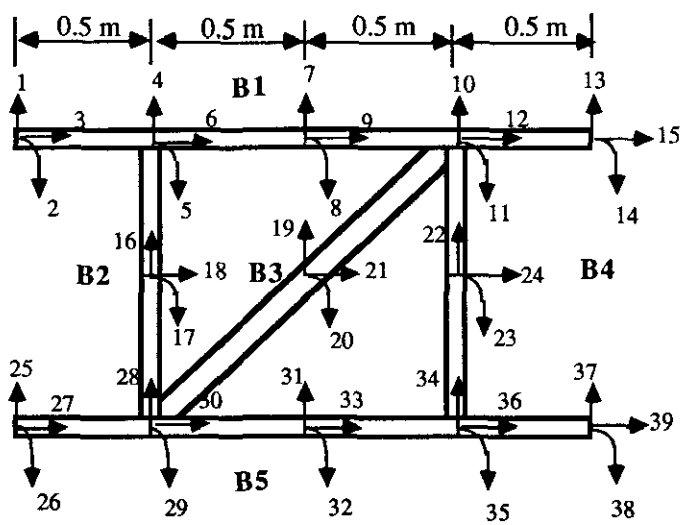


Figure 1. Plane frame model

	B1	B2	B3	B4	B5
EI (Nm^2)	21333	5208	14292	9000	9000
M (kg/m)	6.280	3.925	5.495	4.710	4.710
EI_i (Nm^2)	25000 (5000)	6000 (500)	10000 (3000)	11000 (2000)	11000 (2000)
M_i (kg/m)	6.0 (0.5)	3.5 (0.5)	5.0 (0.5)	5.5 (0.5)	5.5 (0.5)

Table 1. Simulated and initial model parameters

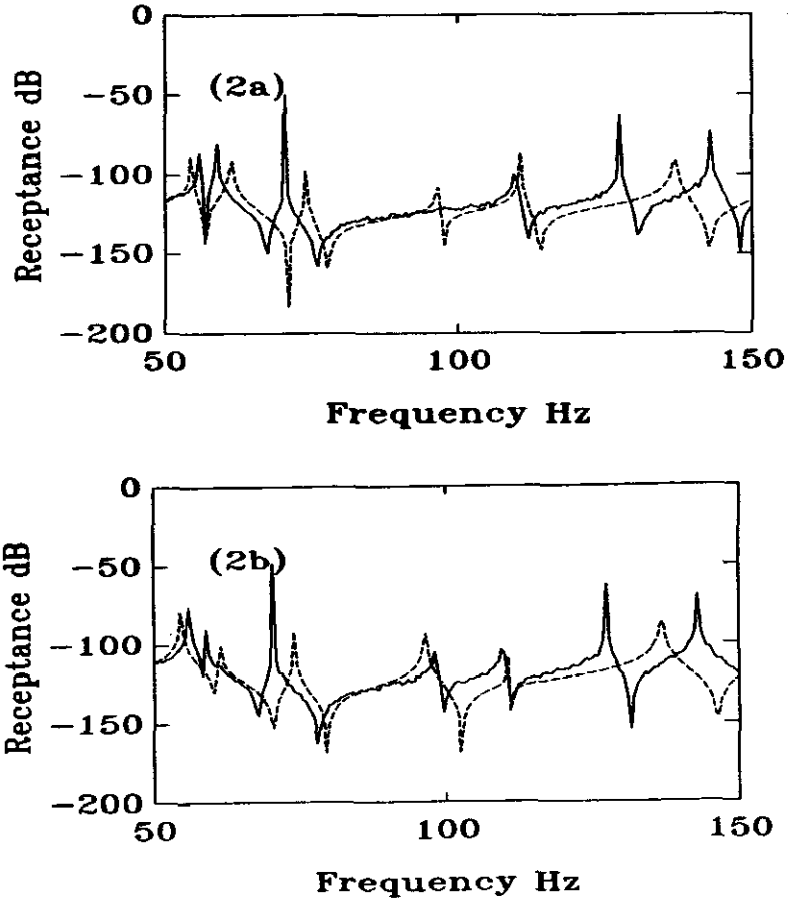


Figure 2. Receptance at the excitation coordinate
a). Coord. 24 b) Coord. 31
------- initial model
_____ simulated frame.

	B1	B2	B3	B4	B5
EI_U (Nm^2)	21373	5271	14280	9065	9040
M_U (kg/m)	6.282	3.974	5.502	4.731	4.735

Table 2. Updated parameters

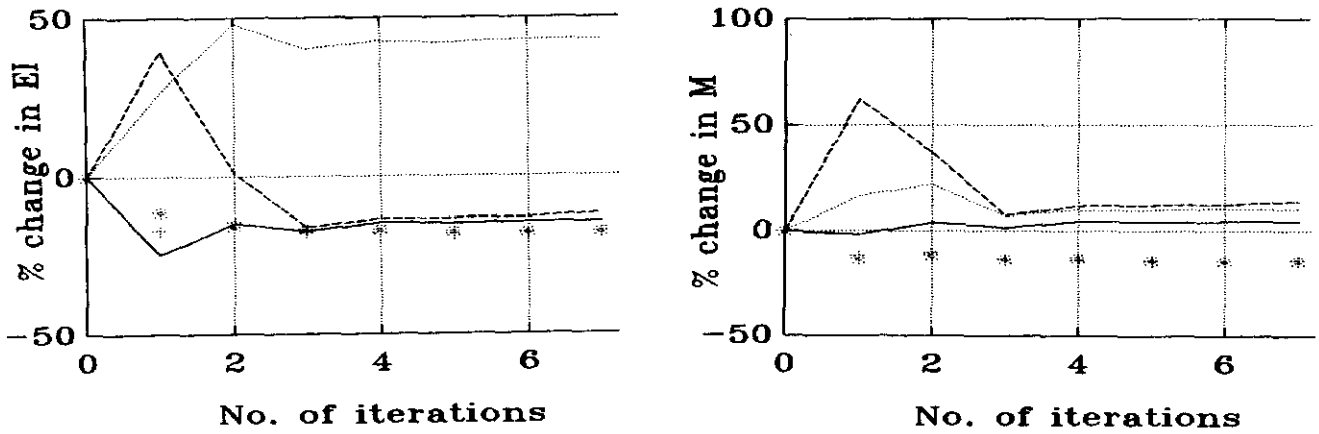


Figure 3. Convergence of the stiffness and mass parameters
a) stiffness *EI* b) mass *M*
___ B1, ---- B2, ... B3, + + + B4, * * * B5.

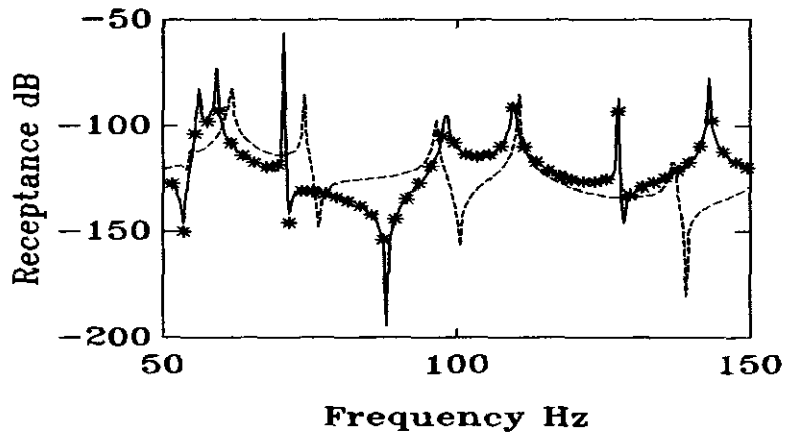


Figure 4. Rotational receptance prediction at coordinate 11 *for excitation at coordinate 31;*
_____ simulated frame
----- initial model
* * * updated model