On the bieberbach and koebe constants of a simply connected domain

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Abstract
The classical Bieberbach coefficient estimate and Koebe 1/4-theorem for univalent functions in the unit disk may be formulated, in terms of a naturally defined Bieberbach constant $n(D)$ and Koebe constant $C(D)$, as $n(D)\leq 2$ and $C(D) \geq 1/4$ for each simply connected domain D. We show that these inequalities have the same extremal domains, by means of a variation on the classical argument that yields the Bieberbach and Koebe Theorems, and describe how this is related to work of Flinn and Herron and of Pommerenke. Such extremal domains satisfy $n(D)C(D)=1/2$. An example of a simply connected domain for which $n(o{:}D)C(D)\neq 1/2$ is constructed, thereby proving that equality does not always hold in the inequality $n(D)C(D) \geq ½$, due to Osgood. In passing, we raise an interesting question on the relationship between the second coefficient of a univalent function f and the radius of the largest disk about $f(0)$ that is covered by f

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