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A Mixed Convection Boundary Layer Flow over a Vertical Wall in a Porous Medium, with Exponentially Varying Fluid Viscosity

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Abstract

This study investigates a mixed convection boundary layer flow over a vertical wall embedded in a highly porous medium. The fluid viscosity is assumed to decrease exponentially with temperature. The boundary layer equations are transformed into a non-similar form using an appropriate non-similar variable ξ and a pseudo-similar variable η. The non-similar equations are solved using an efficient local non-similarity method. The effect of viscosity variation parameter on the heat transfer, skin friction and the velocity and temperature distribution within the boundary layer is investigated. The viscosity variation parameter, the viscous dissipation parameter and non-similarity variable are shown to have a significant effect on velocity and thermal boundary layer and also on the skin friction coefficient and heat transfer at the wall.

Keywords

Porous Medium, Non-Darcy, Mixed Convection Boundary Layer, Temperature Dependent Viscosity

1. Introduction

Mixed convection boundary layer flow through a porous medium is of primary importance due to its applications in industry such as geothermal operations (for example dynamics of hot springs), terrestrial heat flow through an aquifer, flow of moisture through porous industrial material etc. Engineering applications include porous thermal insulation and heat exchangers with fluidized beds etc.

Reviews on convective heat transfer in porous medium are well documented in the books by Nield and Bejan, [1] and Pop and Ingham [2], Kaviany [3] and in the handbook of porous media edited by Vafai [4]. Numerous studies on mixed convection boundary through a porous media using the Darcy model and extended Darcy

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models have been carried out, by others, by Karabis *et al.* [5] investigated a mixed forced-free convection boundary layer flow over a vertical wall in a highly porous medium, Chen [6] who looked at a boundary layer flow over a horizontal in a saturated porous medium. Using the model developed by Vafai and Tien [7], Hong *et al.* [8] investigated the non-Darcian effects on a flow over a vertical plate in porous media.

Most fluids used in engineering have temperature varying viscosity. A study by Hossain and Munir [9] investigated a mixed convection boundary layer with temperature dependent viscosity where it was shown that the viscosity variation parameter has an effect on the wall skin friction coefficient and wall heat transfer coefficient. Jayanthi and Kumari [10] studied the effects of variable viscosity on a non-Darcy flow of a non-Newtonian fluid over a vertical surface in porous medium. They showed that variable viscosity parameter has a significant effect on the fluid flow properties.

The current study focuses on a boundary layer flow over a vertical wall embedded in a highly porous medium and with variable viscosity. The non-similar boundary layer equations are solved using the local non-similarity method due to Sparrow *et al.* [11] and Sparrow and Yu [12]. In Section 2 the mixed convection boundary layer flow equations are formulated and transformed to the non-similar form, in Section 3 the equations are solved and the results presented and discussed in Section 4. Conclusions are drawn in Section 5.

2. Mixed Convection Boundary Layer Flow in a Porous Medium

A steady, two-dimensional flow of a viscous incompressible fluid over a heated semi-infinite vertical wall embedded in a highly porous medium is studied. The x-axis is taken along the wall and the y-axis as the normal axis to x-axis. The free-stream velocity and temperature are taken as U_∞ and T_∞ respectively and the wall temperature as $T_w\left(>T_\infty\right)$. The fluid viscosity μ is assumed to decrease exponentially with temperature. Under the boundary layer approximation for a flow through a porous medium and the Boussinesq's approximation, the mixed convection boundary layer equations through a porous media take the form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\rho_0\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho_0 U_\infty\frac{\mathrm{d}U_\infty}{\mathrm{d}x}+g\rho_0\alpha\left(T-T_\infty\right)+\frac{\tilde{\mu}}{K}\left(U_\infty-u\right)+\frac{\partial}{\partial y}\left(\tilde{\mu}(T)\frac{\partial u}{\partial y}\right) \tag{2.2}$$

$$\rho_0 c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2}+\tilde{\mu}(T)\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

The appropriate boundary conditions for the flow are:

$$u\left(x,0\right)=v\left(x,0\right)=0,\quad T\left(x,0\right)=T_w$$
$$u\left(x,\infty\right)=U_\infty\left(x\right),\qquad T\left(x,\infty\right)=T_\infty$$

Here g is the gravitational acceleration, α is the coefficient of thermal expansion, κ is the thermal conductivity, c_p is the specific heat at constant pressure, ρ_0 is the fluid density at reference temperature T_0 and K is the permeability of the porous medium. The wall temperature T_w is assumed to be a constant. Most common fluids have dynamic viscosity that varies exponentially with temperature. Here we use Arrhenius model which takes the exponential form

$$\tilde{\mu}(T)=\mu_0 e^{-\gamma(T-T_\infty)}$$

where μ_0 is reference viscosity at reference temperature T_0 and γ is a viscosity variation number defined as

$$\gamma=\frac{1}{T_w-T_\infty}\ln\left(\frac{\mu_0}{\mu_w}\right)$$

where $\mu_0>\mu_w$ with be μ_w being the fluid viscosity at the wall temperature T_w. For gases $\gamma<0$ and for liquids it is positive. For small values of γ Taylor expansion leads to the linear or the inverse relation of viscosity with temperature.

Non-Similar Boundary Layer Equations

The equations are rendered non-dimensional by introducing a boundary layer pseudo-similarity variable $\eta(x,y)$ and a non-similar variable $\xi(x)$, defined respectively as

$$\eta(x,y)=\frac{y}{x}\sqrt{Re_x} \qquad \xi(x)=\frac{Gr_x}{Re_x^2}$$

with Gr_x and Re_x being the local Grash of number and the local Reynolds number, defined as

$$Gr_x=\frac{g\alpha(T_w-T_\infty)x^3}{\nu_0^2},\ Re_x=\frac{U_\infty x}{\nu_0}$$

where $\nu_0=\dfrac{\mu_0}{\rho_0}$ is the kinematic viscosity.

We use the scaling

$$u=U_\infty(x)f'(\eta,\xi),\quad T-T_\infty=(T_w-T_\infty)\theta(\eta,\xi),\quad \mu(\theta)=e^{-\varepsilon\theta},$$

where $\mu(\theta)=\tilde{\mu}/\mu_0$ is the dimensionless viscosity and $\varepsilon=\gamma(T_w-T_\infty)$.

From the equation of continuity:

$$v=-\frac{U_\infty}{\sqrt{Re_x}}\left[\frac{\beta+1}{2}f+\frac{\eta(\beta-1)}{2}f'+\xi(1-2\beta)f_\xi\right]$$

with

$$\beta=\frac{x}{U_\infty}\frac{\mathrm{d}U_\infty}{\mathrm{d}x}$$

Assuming power law variation in the free-stream velocity $U_\infty=Ax^n$, we have $\beta=n$. The non-similar boundary layer equations take the form:

$$f'''=\varepsilon\theta' f''-S(1-f')+e^{\varepsilon\theta}\left[n\left[(f')^2-1\right]-\frac{n+1}{2}ff''-\xi\theta\right]+e^{\varepsilon\theta}\xi(1-2n)\left[f'_\xi f'-f_\xi f''\right] \tag{2.4}$$

$$\theta''=-\mathrm{Pr}\left[\frac{n+1}{2}f\theta'+Ec(f'')^2e^{-\varepsilon\theta}\right]+\mathrm{Pr}\,\xi(1-2n)\left[f'\theta_\xi-f_\xi\theta'\right] \tag{2.5}$$

where S_x is the dimensionless local permeability parameter, Pr is the Prandtl number and Ec is the local Eckert number, defined respectively as

$$S_x=\frac{\nu_0 x}{KU_\infty},\quad \mathrm{Pr}=\frac{\mu_0 c_p}{\kappa},\quad Ec_x=\frac{U_\infty^2}{c_p(T_w-T_\infty)}.$$

The boundary conditions, in non-dimensional form, are:

$$\begin{aligned}&f'(\xi,0)=0,\quad (n+1)f(\xi,0)+2\xi(1-2n)f_\xi(\xi,0)=0,\quad \theta(\xi,0)=1\\&\lim_{\eta\to\infty}f'(\xi,\eta)=1,\quad \lim_{\eta\to\infty}\theta(\xi,\eta)=0.\end{aligned} \tag{2.6}$$

The shear stress and the heat transfer at the wall can be represented using the local skin friction coefficient and the local Nusselt number, defined by

$$c_f\sqrt{Re_x}=e^{-\varepsilon}f''(\xi,0),\quad Re_x^{-\frac{1}{2}}Nu=-\theta'(\xi,0)$$

3. Numerical Solutions of the Non-Similar Boundary Layer Equations

The system of Equations (2.4)-(2.6) is solved using the non-similar method of Sparrow *et al.* [11]. This method has been used extensively in solving non-similar mixed convection boundary layer equations by among other

Sparrow and Yu [12], Gorla *et al.* [13], Mureithi and Mason [14], Bég *et al.* [15] etc. The method involves approximating the solutions at different levels of truncation. We introduce the notation:

$$f_1 = \frac{\partial f}{\partial \xi},\ f_1' = \frac{\partial f'}{\partial \xi},\ \theta_1 = \frac{\partial \theta}{\partial \xi},\ f_2 = \frac{\partial f_1}{\partial \xi},\ \theta_2 = \frac{\partial \theta_1}{\partial \xi} \text{ etc.}$$

The first level of approximation involves solving the local similarity equations, where all terms involving the ξ and its derivatives (that is the non-similar terms, f_1, θ_1) in the Equations (2.4)-(2.6) are neglected.

At the second level of truncation, the method involves solving the system (2.4)-(2.6) together with the auxiliary system obtained by differentiating (2.4)-(2.6) partially with respect ξ and neglecting all terms involving second order differentiation with respect to ξ (that is f_2, θ_2 etc. The auxiliary system takes the form:

$$\begin{aligned} f_1''' = {} & \varepsilon\left(\theta_1' f'' + \theta' f_1''\right) + S f_1' + \varepsilon e^{\varepsilon\theta}\theta_1\left[n\left(\left(f'\right)^2 - 1\right) - \frac{n+1}{2} f f'' - \xi\theta\right] + \varepsilon e^{\varepsilon\theta}\xi\left(1-2n\right)\left[f' f_1' - f_1 f''\right] \\ & + e^{\varepsilon\theta}\left[2nf f_1' - \frac{n+1}{2}\left[f_1 f'' + f_1' f'\right]\right] - e^{\varepsilon\theta}\left[\xi\theta_1 + \theta\right] + e^{\varepsilon\theta}\left(1-2n\right)\left[f' f_1' - f_1 f'' + \xi\left[\left(f_1'\right)^2 - f_1 f_1''\right]\right] \end{aligned} \tag{2.7}$$

$$\frac{1}{\Pr}\theta_1'' = -\frac{n+1}{2}\left(f_1\theta' + f\theta_1'\right) - Ece^{-\varepsilon\theta}\left[2f f_1'' - \varepsilon\left(f''\right)^2\theta_1\right] + \left(1-2n\right)\left[f'\theta_1 - \theta' f_1 + \xi\left[f_1'\theta_1 - f_1\theta_1'\right]\right] \tag{2.8}$$

$$f_1'\left(\xi,0\right) = 0,\ \left[3\left(1-n\right)\right] f_1\left(\xi,0\right) = 0,\ \theta_1\left(\xi,0\right) = 0,\ f_1'\left(\xi,\infty\right) = \theta_1\left(\xi,0\right) = 0 \tag{2.9}$$

The system (2.4)-(2.9) solved as if it is an ordinary differential system, with ξ being treated as a parameter. The results obtained are a better approximation than those obtained at the first level of approximation.

An even better approximation can be obtained at the third order level of approximation, where the system (2.4)-(2.9) are solved together with an of auxiliary system obtained by further differentiating Equations (2.7)-(2.9) partially with respect to ξ and all terms containing third order partials with respect to ξ are neglected.

4. Discussion of Results

We approximate the solution of (2.4)-(2.6) using the second level of approximation. The system of Equations (2.4)-(2.9) is solved using the fourth order Runge-Kutta method combined with a Newton root refinement scheme. Computations were carried out for the case when $\Pr = 0.7$ (air) and viscosity that increases exponentially with increase in temperature $\left(\varepsilon < 0\right)$.

Figure 1 show the effect varying the non-similar parameter ξ on the scaled stream-wise velocity, f', and viscosity, $\mu\left(\theta\right)$, distribution in the boundary layer. At a fixed stream-wise location, ξ becomes a buoyancy-related parameter. As ξ increases velocity overshoots (exceeding free-stream velocity) are observed within the boundary layer. The maximum values of the super-velocities increase with increase in ξ Results in figure 1also show that increasing ξ has the effect of decreasing the thickness of the viscosity boundary layer.

The effect of varying the viscous dissipation parameter Ec was investigated and the results are presented in **Figure 2**. The results indicate that as Ec increases, the amplitude of the super-velocities increase. Temperature is also shown to increase with increase in Ec and for high enough values of Ec (in this case $Ec \geq 0.6$) there is a small region adjacent to the wall, where temperature exceeds its wall value of one. It follows that viscosity function also has positive gradient in that small region. This is an indication that increasing Ec results in increased heat transfer from the wall to the fluid.

Figure 3 shows stream-wise velocity and viscosity distributions within the boundary layer for different values of ε. The results show that at a fixed value of ξ velocity overshoots increase with increase in ε. On the contrary, viscosity decreases with increase in ε.

Heat transfer at the wall (represented by the Nusselt number, $-\theta\left(\xi,0\right)$ and shear stress at the wall (represented by skin friction coefficient, $\mathrm{e}^{-\varepsilon} f''\left(\xi,0\right)$ were investigated as functions of the effect of ξ and various values of ε **Figure 4** shows that the skin friction coefficient and Nusselt number increase with increase in ε. The skin friction coefficient increases linearly with ξ while the Nusselt number decreases with increase in ξ.

Similar results showing the effect of Ec on both the skin friction and the Nusselt number are presented in **Figure 5**. The results show that the skin friction increases linearly with increase in ξ while the Nusselt number decrease with increase in Ec. On the other hand, increase Ec results in increase in the skin friction coeffi-

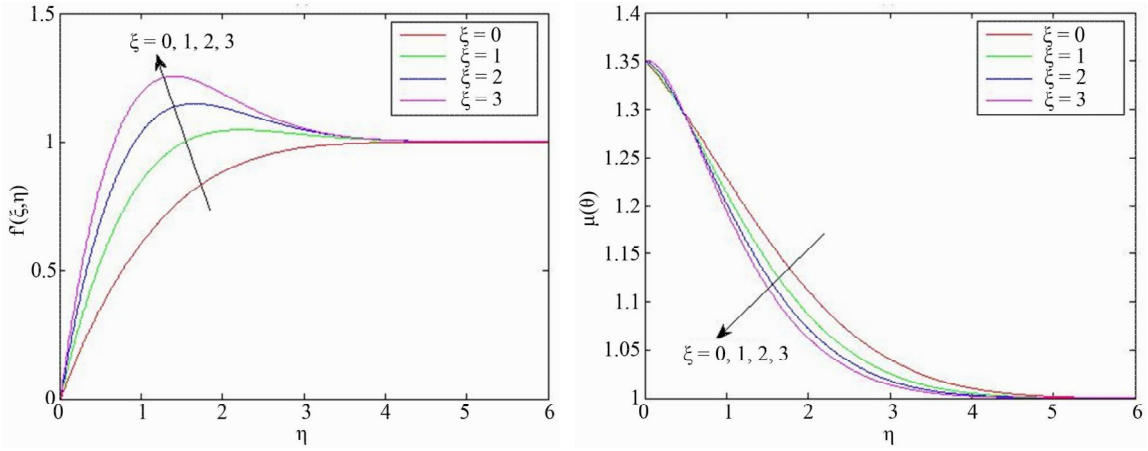
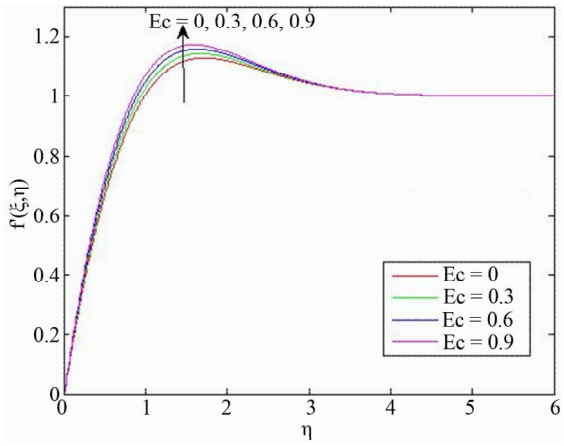


Figure 1. The effect of varying ξ on (a) scaled stream-wise velocity f' and (b) viscosity $\mu(\theta)$ for the case when $Ec = 0.4$, $S = 0.5$, $n = 0.2$, $\varepsilon = -0.3$.



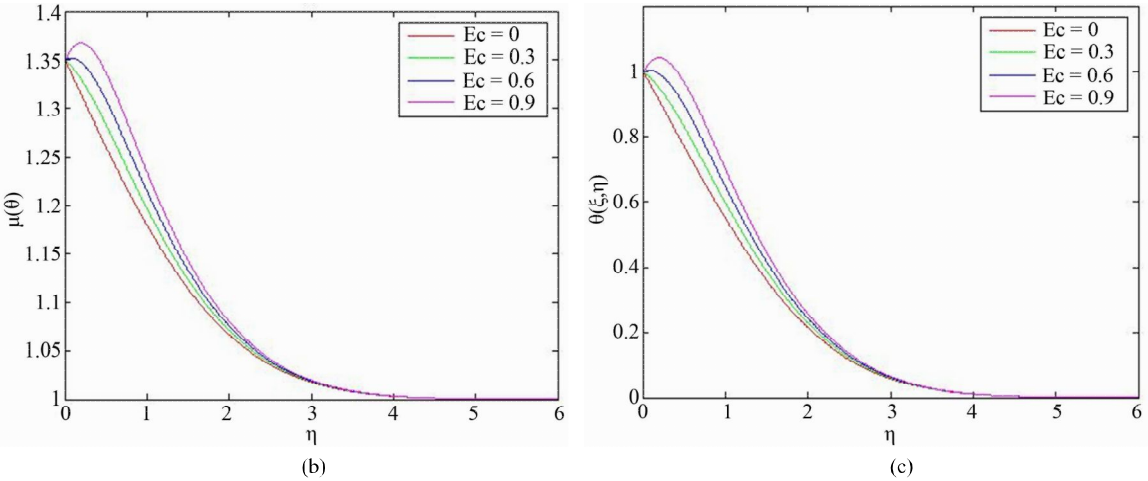
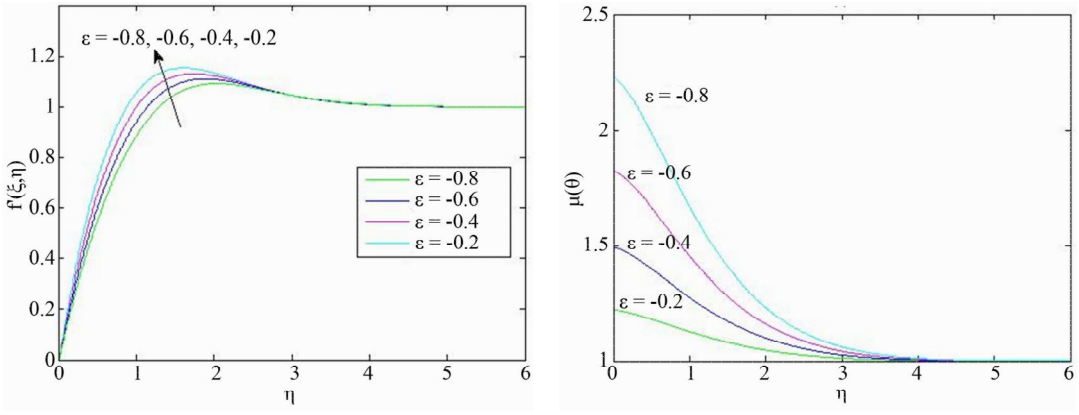
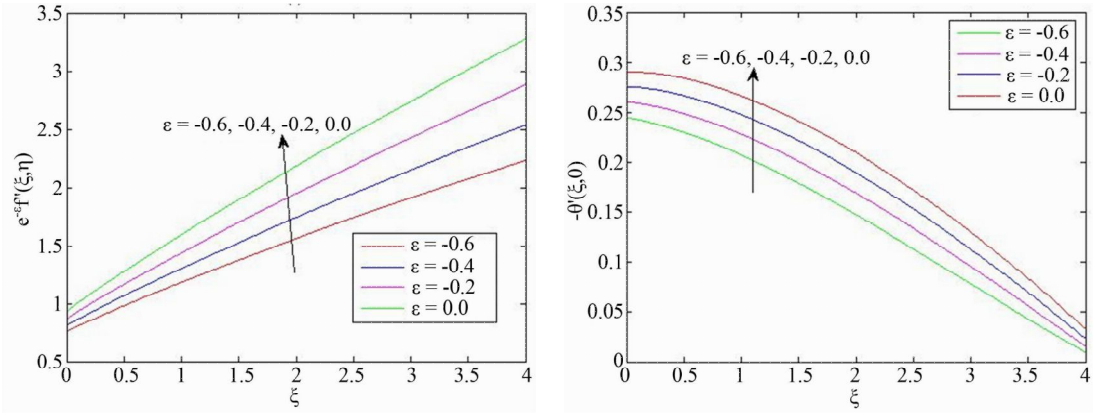


Figure 2. Effect of varying Ec on (a) $f'(\xi,\eta)$, (b) $\mu(\theta)$ and (c) $\theta(\xi,\eta)$ for the case when $\varepsilon = -0.3, S = 0.5, n = 0.2$, $\xi = 2$.



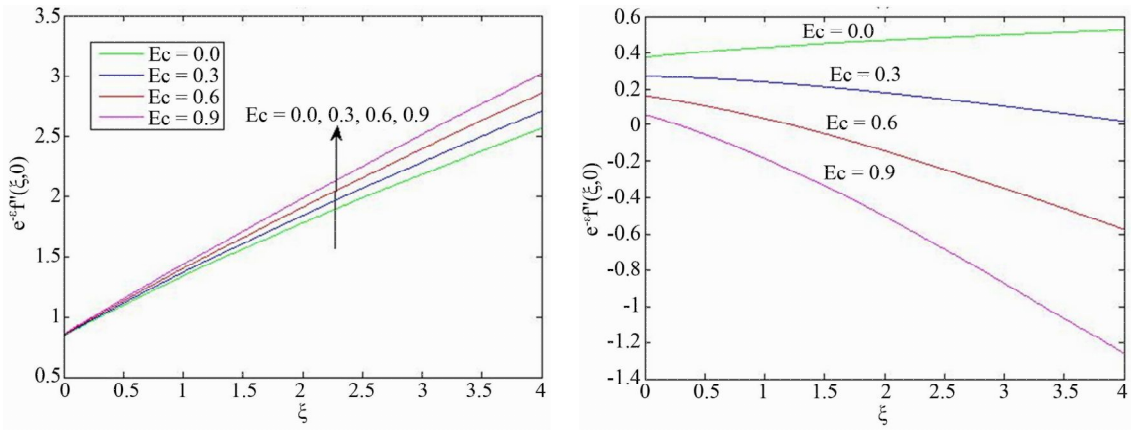
(a) (b)

Figure 3. Effect of varying viscosity variation parameter ε on (a) f' and (b) $\mu(\theta)$ for the case when $Ec = 0.4$, $S = 0.5, \xi = 2, n = 0.2$.



(a) (b)

Figure 4. The effect of varying ξ and ε on the (a) skin friction coefficient and (b) Nusselt number for $Ec = 0.4$, $S = 0.5$, $n = 0.2$.



(a) (b)

Figure 5. The variation of (a) skin friction coefficient and (b) Nusselt number with ξ and Ec for $S = 0.5$, $n = 0.2$, $\varepsilon = -0.3$.

cient and a decrease in the Nusselt number. The results for the heat transfer coefficient indicate that at a fixed value of $Ec > 0$ there is a critical value of ξ beyond which the heat transfer coefficient changes sign from positive (heat flows from fluid to the wall) to negative (heat flows from wall to the fluid in the boundary layer).

5. Conclusions

A non-Darcy mixed convection boundary layer flow has been investigated. The results have shown that the non-similarity parameter ξ, which at particular stream-wise location represents thermal buoyancy, has the effect of accelerating the scaled free-stream velocity, resulting in super-velocities within the boundary layer. This effect is exacerbated by increasing viscous dissipation parameter and the viscosity variation parameter. The temperature dependent viscosity decreases with increase in buoyancy-related parameter and viscosity variation parameter but increases with increase in the viscous dissipation parameter.

The skin friction coefficient increases with increase in the viscosity variation parameter, the viscous dissipation parameter and the buoyancy-related parameter. For flows with viscous dissipation effects, there is a critical value of the buoyancy related parameter, above which the Nusselt number changes sign from positive (wall extracts heat from fluid) to negative (fluid extracts heat from wall).

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