Available online at http://scik.org
J. Math. Comput. Sci. 7 (2017), No. 4, 677-698
ISSN: 1927-5307

# STEADY DIVIDEND PAYMENT AND INVESTMENT FINANCING STRATEGY: A FUNCTIONAL MEAN REVERSION SPEED APPROACH

ADELINE PETER MTUNYA[1,3,*], PHILIP NGARE[2], YAW NKANSAH-GYEKYE[1]

[1]School of Computational and Communication Science and Engineering,
Nelson Mandela African Institution of Science and Technology, Arusha, Tanzania
[2]School of Mathematics, University of Nairobi, Nairobi, Kenya

[3]Department of Mathematics, Mkwawa University College of Education, Iringa, Tanzania

Copyright © 2017 Mtunya, Ngare and Nkansah-Gyekye. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

**Abstract.** We study how corporate firms' management can satisfy the shareholders by steady and growing dividend payouts while financing investment growth from the profit. We set the percentage of the profit which is optimal for steady dividends and the rest to be allocated for investment and dividend buffer account. A mean reversion stochastic differential equation with investment function in the drift term has been used to find the optimal dividend and retainment levels. One of the findings shows that for each level of profit there exists a percentage which is optimal for paying steady dividends while financing investment growth. Also we find that having low interest rates is favourable for the strategy of paying steady dividend with investment growth. Moreover, we compared the proposed strategy with a situation of steady dividend without investment and found that the strategy with investment is more appropriate as it gives more values to the shareholders. In addition we find that the exponential and linear responses of the investment function on investment amount give out the same results. Companies in developing economies should consider steady and growing dividends as they expand their investments, while policies of such economies should enforce low interest rates and influence companies to pay dividends.

**Keywords:** firm investment; internal financing; dividend policy; developing economies; functional mean reversion speed; stochastic optimal control.

**2010 AMS Subject Classification:** 91G50.

*Corresponding author
E-mail address: mtunyaa@nm-aist.ac.tz, tunyason@yahoo.co.uk
Received March 21, 2017

# 1. Introduction

There have been a number of comprehensive theoretical and empirical studies on dividends but dividend policy and its determinant remains a puzzle to be fixed in corporate finance [1, 2, 3]. Maintenance of the dividend payout level is a priority on par with the investment decisions of the firms' financial managers [4]. Corporate shareholders demand cash dividends, therefore, managers have to equilibrate between different interests of the shareholders so that they could utilize available investment profitable opportunities and pay the required dividend [5, 2]. Dividend policy has been one of the most critical corporate decisions from both the management and the shareholders [6, 7]. Generally, firm managers do consider sustaining stable or increased earnings for maintaining steady dividend payouts [4, 3]. The trade-off between retention for investment and dividend distribution evolves over time as profits accumulate and investment opportunities decline [6]. Myers and Majluf's Pecking-Order points out that firms' managers prefer retained earning over other sources such as debt and equity when planning to finance business expansion [8]. However it is argued by [9] that corporates in developed countries relied more on internally generated fund while their counterparts in developing countries relied more on externally generated fund such as the bank loans. Therefore, significant differences exist between the developed and developing countries regarding financing choice of firms for investment [9]. This is to say that the firm manager's preference for retained earning as stated by [8] is not favoured in developing economies. Thus there is a need to come up with a strategy that will help managers fulfill the demand for cash dividend from the shareholders meanwhile fulfilling their preference for internal financing for investment growth.

This study aims at establishing a strategy that can be adopted by firm managers in order to optimally equilibrate the steady and high dividend payment and the amount for investment from the collected revenue. By steady dividend payment we refer to a series of dividend payments in which the payments orderly increase or at least maintains the previous level. The amount for investment on the other hand is the retained cash from the profit, after dividends are settled, that is used for business expansion. Despite the fact that there are several studies that present mathematical models on dividends and investment, unluckily we could not come across one in the literature that provides for steady dividend payment with investment being financed from the

profit. Our contribution from this study is essentially a milestone from one of our previous studies [10] in which we studied the steady dividend payment without considering the investment. By consideration of the investment expansion in the model, not only improves the result but also complicates the analysis as managers have to find an optimal balance between retainment for investment and steady dividend.

Today there exists a pool of literature in financial mathematics and financial economics about optimal dividend policies and investment strategies. History records some prominent studies by Lintner [11], Miller and Modigliani [12], Gordon [13], Black [1] and, Jeanblanc-Picqu and Shiryaev [14]. John Lintner (1956) was the first to state that firms try to maintain stability of the dividend policy and that dividends are indicators of the future performance [3]. This was quite before the popular advocacy by Miller and Modigliani that dividend policy is irrelevant in determining the firm's value [12]. However, Gordon through his 'bird in hand' theory argues that the shareholders do prefer dividend payment over capital gains so as to minimize the risk of investment [13]. Gordon proposed a mathematical model that shows the dependence of the market value of the share on the distributed dividends. There is a good number of empirical studies that have been conducted over the Gordon's results [15, 16, 17]. For example, [17] found out that the market-to-book value are significantly higher for the dividend paying firms as compared to non-paying firms. By running multiple regression they revealed a notable relationship between shareholders wealth and the amount of dividend paid. However, the Gordon model has been criticized for ignoring the business risk which has a direct impact on the firm's value. As a reaction on the business risk, [14] considered the model by Stochastic Differential Equation (SDE) of [18] (a preprint by the time) on dividend strategy and analyzed the model by considering some more assumptions such as random moments of the payment and transaction costs. This was a big improvement that has influenced most of the recent studies.

Recent studies have continued to consider modelling by the SDEs or drifted Brownian motion in optimizing dividends and investment strategies of the firms [19, 20, 21, 22]. For instance [21] worked on the problem of finding an optimal control on the dividend and investment strategy of a company. In their study they also involved debt in the balance sheet of the firm. Whereas they regarded debt financing for investment, in this particular study the profit is considered

as the main source to finance investment. In addition to internal financing, we would like to ensure steadiness in the dividend payment which is the problem of firms operating in developing economies. On the other hand [23] studied optimal distribution for a firm in the presence of regime shifts. They considered a company whose profit evolves as a Brownian motion with positive drift that is modulated by a finite state Markov chain. One of their results showed that if the drift is positive in each state, then it is optimal to adopt a barrier strategy at certain regime dependent levels. Similarly, [24] developed numerical methods for finding optimal dividend payout whereby they introduced a singular control formulation of surplus and discounted payoff function. They model surplus by a regime switching process subject to both regular and singular controls. More recently, [25] have studied the optimal control on the dividend and investment policy of a company operating under uncertain environment and risk constraints. They assumed that the company may make investment decision of selling or acquiring assets whose value is governed by a stochastic process. They concluded that when asset prices gets higher, the firm has to hold sufficient amount of cash in order to invest in more expensive assets. It is also preferable to distribute dividend when asset prices are very high and sell the assets when the cash reserve gets near zero in order to avoid bankruptcy. Though this is a good plan for the investment side but could not provide for stable and steady dividend payment as required by shareholders, particularly in developing economies. Moreover, assuming such a high liquidity for the assets in the action of buying and selling might not be a reality in developing economies where the efficiency of financial markets is low. In this study, therefore, we take a different orientation from the referred studies by considering steadiness in the dividend payments and also consider firm investment financing from the same profit.

Significant differences exist in the dividend policies of firms between the developed and emerging markets. Dividends in emerging market economies have been observed to be low meanwhile the companies in the same do not pursue stable dividend policy [26, 27]. This draws the attention to find innovative approaches in managing firms in the developing countries. Moreover, there is a need to pay dividend consistently in terms of the amount and frequency of payment as the shareholders in emerging market economies continuously demand for dividends [28]. However, optimizing in the dividend should not keep aside the need for growth of firms

particularly in developing economies where most of firms need to grow [4, 2]. These facts attract for more studies on dividends and investment policies for firms in developing economies as undertaken in this article.

In this study the investment and dividend levels are modeled in dependence of the profit level because we assume that the decisions to pay dividend and/or invest considers the profit accumulated. The model is actually an SDE from which the optimization by singular control results to an ODE. We characterize the solution of the ODE analytically and use it for the numerical experiments.

The remainder of this article proceeds as follows. Section 2 describes the proposed model, defines the investment function, presents the buffer dynamics, defines the bankruptcy condition and the objective function. In section 3 we provide the proof for the existence, uniqueness and concavity of the value function. This shows that the value function for the objective function can be the solution of the dynamic programming equation. In section 4 we carry out numerical experiments with the discussion of the results. We present plots for general overview of the value function over the profit and dividend axes, determination of the optimal percentage for dividend, sensitivity of the value function on the interest rate, effectiveness of the proposed strategy and investment function response on investment amount. Section 5 is about the summary of the results, concluding statements and suggestion for the way forward.

## 2. Model formulation

We consider a filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathcal{P})$ under common assumptions in describing uncertainty. Let $W_t$ be one dimensional $(\mathcal{F}_t)$ adapted Wiener process. The firm under consideration is said to basically generate profit $Y_t$ whose trend can be mathematically represented by Stochastic Differential Equation (SDE) with a drift parameter $\mu$ and a fixed volatility parameter $\sigma$ as follows

$$dY_t = \mu dt + \sigma dW_t. \tag{1}$$

We denote by $Z_t$ the cumulative amount of dividends paid from zero up to $t$, by $R_t$ the total amount collected as the retainment from the profit up to time $t$, and by $I_t$ the total amount

channeled to the investment from the retainment. The three processes $Z$, $R$ and $I$ are considered to be nondecreasing, $(\mathcal{F}_t)$ - adapted processes and have sample paths which are left-continuous with right limits. In addition we include a functional mean reversion speed $\phi$ which virtually acts as the force that brings back the profit level to its mean by shifting a reasonable amount of cash to and from the reserved profit (RP). The function $\phi$ is dependent on the amount of dividend required and the profit made at a given time. On the other hand we consider the impact of investment on the profitability of the firm in which we incorporate the function $\psi$ that depends on the investment amount. In fact $\psi$ is an increasing function in $I_t$ and its value is additive to the profit level $Y$. Thus we have the following model for steady dividend and investment

$$dY_t = \phi(Y_t, Z_t)[\mu - (Y_t + \psi(I_t))]dt + \sigma dW_t - dZ_t - dR_t,\ Y_0 = y; \tag{2}$$

$$I_t = (1-\delta_t)R_t, \quad \delta_t \in (0,1] \tag{3}$$

where $\delta_t$ is the fraction of $R_t$ that remains in RP after allocating amount for investment at time $t$. The variable $\delta$ is evaluated in such a way that more fund is used for investment when the profit is higher far from the required dividend amount. So $\delta_t$ can have the following mathematical representation

$$\delta_t = \frac{Z_t}{Y_t}, \quad Y_t \geq Z_t \tag{4}$$

which indicate that if the profit is the same as the required dividend then there is no amount for investment. In this case we consider $\psi$ as an exponential function of $I_t$ as follows

$$\psi(I_t) = \begin{cases} \alpha e^{(1-\delta_t)R_t}, & \text{if } Y_t \geq Z_t, \\ 0, & \text{if } Y_t < Z_t \end{cases} \tag{5}$$

where $\alpha$ is a constant. If the function $\psi$ is to respond to the interest rate $r$, which is taken to be constant in this study, then we have

$$\psi(I_t) = \begin{cases} \alpha e^{(1-\delta_t)R_t - r}, & \text{if } Y_t \geq Z_t, \\ 0, & \text{if } Y_t < Z_t. \end{cases} \tag{6}$$

Alternatively, it is possible for the increasing function $\psi$ to assume a linear dependence on $I_t$ as follows

$$\psi(I_t) = \begin{cases} m(1-\delta_t)R_t + c, & \text{if } Y_t \geq Z_t, \\ 0, & \text{if } Y_t < Z_t \end{cases} \tag{7}$$

where $m$ and $c$ are constants. In section 4 we present a comparison of the results between the exponential and linear response of $\psi$ on $I_t$.

We denote the RP by $B$, that is, the profit buffer whose increments or decrements depend on the difference between the profit $Y$ and the anticipated dividend amount $Z$ with the possible investment amount $I$ at a given time instance. The plan for the dividend as described in [10] is such that always the dividend payouts will increase by the percentage $\gamma$ when profit is sufficient to pay the dividend otherwise the dividend retains its previous level. When $Y_t - Z_t \geq 0$ at time $t$, the profit buffer $B_t$ is increased by the value $\delta_t R_t$ and when $Y_t - Z_t < 0$, a sum amounting $|Y_t - Z_t|$ is taken from the profit buffer and being added to the profit level $Y_t$ in order to satisfy the anticipated dividend $Z_t$. Therefore, the increments or decrements on the dynamics of the profit buffer at any time are expressed as

$$\Delta B_t = \delta_t R_t \mathbb{1}_{(Y_t - Z_t \geq 0)} + (Y_t - Z_t)\mathbb{1}_{(Y_t - Z_t < 0)}. \tag{8}$$

The mean reversion speed $\phi(Y_t, Z_t)$ will always become faster and faster as the increments $\Delta B_t$ assume absolutely greater and greater values. As in one of our previous study [10] we consider an exponential relationship between $\phi(Y_t, Z_t)$ and $\Delta B_t$, and therefore we have the functional mean reversion speed given by

$$\phi(X_t, Z_t) = e^{\Delta B_t}. \tag{9}$$

We assume that the survival of the company relies mostly on its ability to satisfy the shareholders by supplying dividend. So the company should have positive profit values or have sufficient amount in the profit buffer for its existence. We then define the bankruptcy time $\tau$ by

$$\tau = \inf\{t \geq 0 : Y_t < 0 \text{ and } B_t < 0\}. \tag{10}$$

Our objective is to find an optimal approach that can be applied by the firms' management in order to ensure stead dividend payments meanwhile allocating fund for furthering investment from the profit. The firm wants to invest as more as possible but is facing demand for cash dividends from the shareholders. Given as initial condition the value of the profit $y$, we denote the set of all admissible dividend and the retainment scheme $(Z,R)$ by $\mathcal{A}(y)$. From equation (3) it is reveled that the admissibility of the investment $I$ is subject to the admissibility of the retainment $R$. Thus our problem on steady dividend and investment is to maximize the value function $J$ with the representation

$$J(y;Z,R) = \mathbb{E}^y \left[ \limsup_{t\to\infty} \left( \int_0^{t\wedge\tau} e^{-rt} dZ_t + \int_0^{t\wedge\tau} e^{-rt} dR_t \right) \right], \tag{11}$$

where the discount factor is the constant interest rate $r$. The corresponding optimal value function is then defined as

$$v(y) = \sup_{(Z,R)\in\mathcal{A}(y)} J(y;Z,R) \tag{12}$$

and the optimal steady dividend and retainment for investment policy $(Z_t^*, R_t^*)$ is such that

$$J(y;Z_t^*,R_t^*) = v(y). \tag{13}$$

## 3. The Value Function

In this section we analytically handle the mathematical characterization of the optimal value function. Mainly, our target is to maximize the expected discounted dividend payout under steady scheme and the expected discounted retainment amount over all dividend and retainment strategies. The retainment at the end affects the investment which, when undertaken, improves the profit level. Suppose that initially the amount of dividend required is $Z_0 = z$ then if $y - z \geq 0$ the retainment will have the value $R_0 = y - z$. Therefore, the profit buffer is increased by the value $\delta_0 R_0$ and the initial investment has the value $I_0 = (1-\delta_0)R_0$. Otherwise, the profit buffer is reduces by the value $|y-z|$ which implies that the buffer should start with a value greater than $|y-z|$ i.e., $B_0 > |y-z|$. In this case the initial investment becomes zero i.e. $I_0 = 0$ because there is no profit retainment.

From the standard theory of singular control we find the following form of Dynamic Programming equation

$$\max\{\mathcal{L}v(y) - rv(y), -v'(y) + 1, v'(y) - 1\} = 0, \quad y > 0 \tag{14}$$

with the boundary condition $v(0) = 0$ and the operator $\mathcal{L}$ is defined by

$$\mathcal{L}v(y) := \phi(y,z)[\mu - (y + \psi(I_t))]v'(y) + \frac{1}{2}\sigma^2 v''(y) \tag{15}$$

where $I_t$ is as given in equation (2).

Taking into consideration the time value of money, we claim that it is optimal to retain some portion of profit when the profit is more than the required dividend amount. The portion of the retainment that can be allocated for the investment will depend on how far is the profit higher than the required dividend. Moreover, it is optimal to boost up the profit by a deduction from the profit buffer only when profit is less than required dividend. The boosting of profit from the buffer is implemented at most to reach the level of required dividend. No action should be taken by the firm managers given that the profit made equals the required dividend.

For our mathematical analysis, we consider the non-negative solutions of equation (14) in the space $C^2$ as in one of our recent study [10] and also in the study by [29]. Actually the Hamilton-Jacobi-Bellman (HJB) equation (14) is characterized by

$$v'(y) - 1 = 0, \quad \text{for } y \neq z, \tag{16}$$

$$\mathcal{L}v(y) - rv(y) = 0, \quad \text{for } y = z, \tag{17}$$

$$v(0) = 0. \tag{18}$$

The following theorem depicts the important characteristics of the optimal value function

**Theorem 3.1.** *There exists a unique and concave solution for the Dynamic Programming equation* (14)*, and the solution is such that for every* $y > 0$*,*

$$\mathcal{L}v(y) - rv(y) \geq 0 \textit{ for } y = z, \tag{19}$$

$$v'(y) \leq 1 \textit{ for } y < z, \tag{20}$$

$$v'(y) \geq 1 \textit{ for } y > z. \tag{21}$$

**Proof.** We consider the characterization equations (16) to (18). The general solution of the ODE (17) has the following form

$$v(y) = Ae^{\omega_1 y} + Be^{\omega_2 y}, \tag{22}$$

where $A$ and $B$ are fixed real numbers, and $\omega_1$, $\omega_2$ are also real numbers such that

$$\omega_1 = \frac{1}{\sigma^2}\left(-\phi(y,z)[\mu-(y+\psi(I_t))] + \sqrt{(\phi(y,z)[\mu-(y+\psi(I_t))])^2+2\sigma^2 r}\right), \tag{23}$$

$$\omega_2 = \frac{1}{\sigma^2}\left(-\phi(y,z)[\mu-(y+\psi(I_t))] - \sqrt{(\phi(y,z)[\mu-(y+\psi(I_t))])^2+2\sigma^2 r}\right). \tag{24}$$

We learn that $\omega_1 > 0 > \omega_2$. In order to show that the optimal value function $v$ as it appears in equation (17) actually exists, we find the Wronskian of $e^{\omega_1 y}$ and $e^{\omega_2 y}$ found in equation (22) as below

$$\omega_2 e^{\omega_1 y}\cdot e^{\omega_2 y} - \omega_1 e^{\omega_1 y}\cdot e^{\omega_2 y} = \frac{2}{\sigma^2}\sqrt{(\phi(y,z)[\mu-(y+\psi(I_t))])^2+2\sigma^2 r}\cdot e^{(\omega_1+\omega_2)y} \neq 0. \tag{25}$$

Therefore we find that the solution to equation (17) as presented in (22) exists as the Wronskian value is nonzero. Next we show that this solution is unique by considering the fact that for a function to satisfy equation (16) and (17) must be of the form

$$v(y) = \begin{cases} Ae^{\omega_1 y} + Be^{\omega_2 y}, & \text{if } y = z, \\ y - z + Ae^{\omega_1 z} + Be^{\omega_2 z}, & \text{if } y \neq z. \end{cases} \tag{26}$$

In order to evaluate the free boundary value $z$ and, the constants $A$ and $B$, we consider equation (18) while assuming that $v$ is $C^2$ at $z$, as proposed by the smooth pasting condition of singular control. We therefore have the following system of equations

$$Ae^{\omega_1 z} + Be^{\omega_2 z} = z, \tag{27}$$

$$A\omega_1 e^{\omega_1 z} + B\omega_2 e^{\omega_2 z} = 1, \tag{28}$$

$$A\omega_1^2 e^{\omega_1 z} + B\omega_2^2 e^{\omega_2 z} = 0. \tag{29}$$

From the system of equations above we obtain

$$z = \frac{\omega_1 + \omega_2}{\omega_1 \omega_2}, \tag{30}$$

$$A = \frac{-\omega_2 e^{-\omega_1 z}}{\omega_1(\omega_1 - \omega_2)}, \tag{31}$$

$$B = \frac{\omega_1 e^{-\omega_2 z}}{\omega_2(\omega_1 - \omega_2)}. \tag{32}$$

The value of $z$ as it is presented in equation (30) is unique. If we consider the uniqueness of $z$ together with the expressions in the other equations (31) and (32), we find that the constants $A$ and $B$ are also unique. Thus this portrays that the value function $v$ is unique and thus it is established that the Dynamic Programming equation (14) has a unique solution.

In order to show the concavity of the value function, we evaluate the second order derivative of the value function where we have

$$v''(y) = -\frac{\omega_1 \omega_2}{\omega_1 - \omega_2}\left[e^{\omega_1(y-z)} - e^{\omega_2(y-z)}\right] < 0 \text{ for } y \neq z. \tag{33}$$

The inequality results from the fact that $\omega_2 < 0 < \omega_1$. Since $v''(y)$ is negative then $v$ is concave over $y$. The boundary condition $v'(z) = 1$ and $v$ being concave implies the inequalities (19) and (20) in Theorem 3.1. Moreover, considering $v'(y) = 1$ when $y = z$ and that

$$\frac{1}{2}\sigma^2 v''(z) + \phi(y,z)[\mu - (y + \psi(I_t))]v'(z) - rv(z) = 0 \tag{34}$$

leads to equation (19) in the theorem.

The following corollary about the value function is a result of extraction from the proof of the above theorem.

**Corollary 3.1.** *Consider the maximization problem of the value function $J(y;Z,R)$ over all strategies $(Z,R)$ available from $\mathcal{A}(y)$ as presented in* (11) *and* (12)*. Then the concave solution $v$ to the Dynamic Programming equation* (14) *as appears in* (26)*, whereby A, B and z, are constants obtaining values from* (30) *to* (32) *is the optimal value function.*

## 4. Numerical experiments and discussion of results

We provide the illustrations that displays how the optimal value function varies in relation to the profit levels and the anticipated dividend amounts. Finding which level of the dividend as associated with the profit will give the maximum value function, is very important in this case. The numerics are based on the solution of the ODE presented in equation (17). We start by presenting a 3D plot that depicts the behaviour of value function over the profit levels and the planed dividend amounts in a general picture. The second and third figures are about how the value function varies linearly over the dividends and profits for some specific profit levels and dividend levels respectively. These are followed by a table that shows the ratio of the optimal dividend over the profit in relation to the highest possible profit. Then we have other two plots that show the impact of interest rate on investment and how it ultimately affect the value function. The next other two figures offer a comparison of steady dividend and investment approach with a state in which investment is not applied. The last figure is about the differences between the exponential and linear response of the function $\psi$ over the investment amount. Generally, the values of the parameters and initial values used in the numerics have been inherited from the studies by [30] and [31] while the interest rate, the constants $\alpha$, $m$ and $c$, and dividends and profit levels are results of estimations. We set the profit to have extreme value of 3.5 but we allow dividend up to 5.0. This gives us room to investigate what may happen when planned dividend is above profit.

In Figure 1 we observe that the value function increases as the profit level increases. However, it starts by increasing for the low levels of dividend and then eventually decreases over the dividend axis thus creating a ridge of optimal dividend on the surface. This means that there exist some dividend payments which are optimal under the steady dividend and investment strategy.

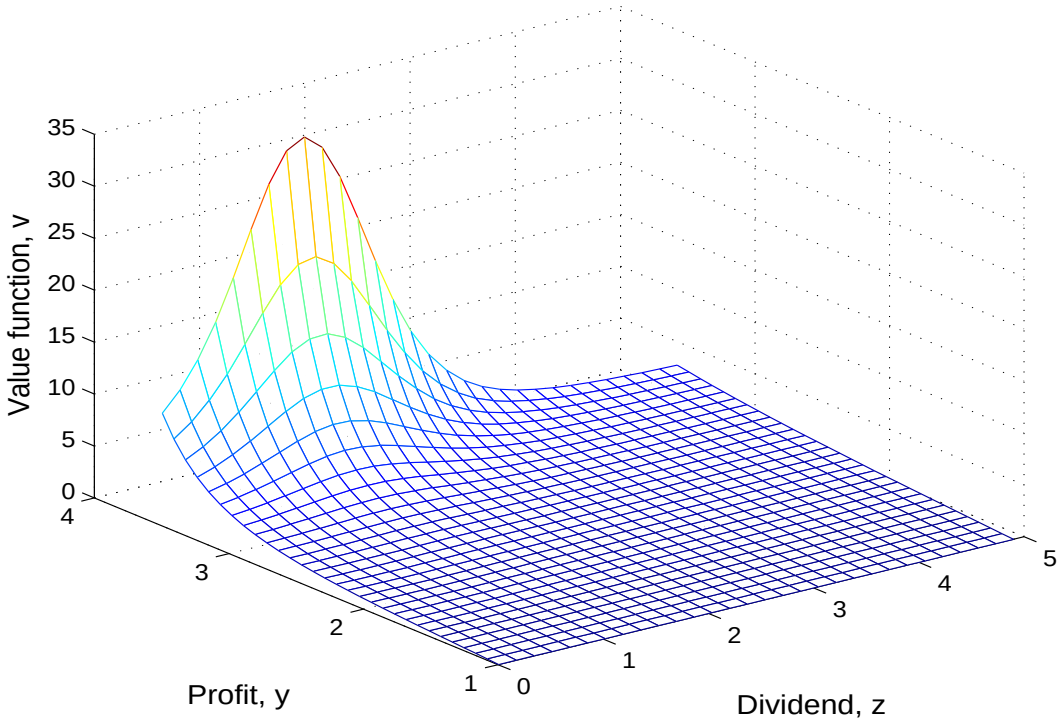



FIGURE 1. Dependence of the value function $v$ over the dividend level $z$ and profit level $y$ with $r = 0.2$, $\mu = 1.5$, $\alpha = 0.02$ and $\sigma = 1.3$.

Figure 2 shows the linear variation of the optimal value function on the dividend axis. We find that there exist localmaxima particularly for higher values of profit. In this case the optimal value function is obtained when the dividend axis marks 1.19. If we relate this value with the profit, which is 2.98, we can conclude that the optimal dividend payment should be 40% of the profit. Table 1 shows the ratios of the optimal dividends over the profits in relation to the highest possible profits. We see in general that the percentage of optimal dividend to the profit made takes values between 36% and 41%. With the assurance that the dividends do not exceed the profit the lowest percentage (i.e., 36%) is generally recommended. Figure 3 shows that the optimal value function increases exponentially over the profit and this approach performs better when the profit is high and the anticipated dividend should not be exhaustive to the profit made.

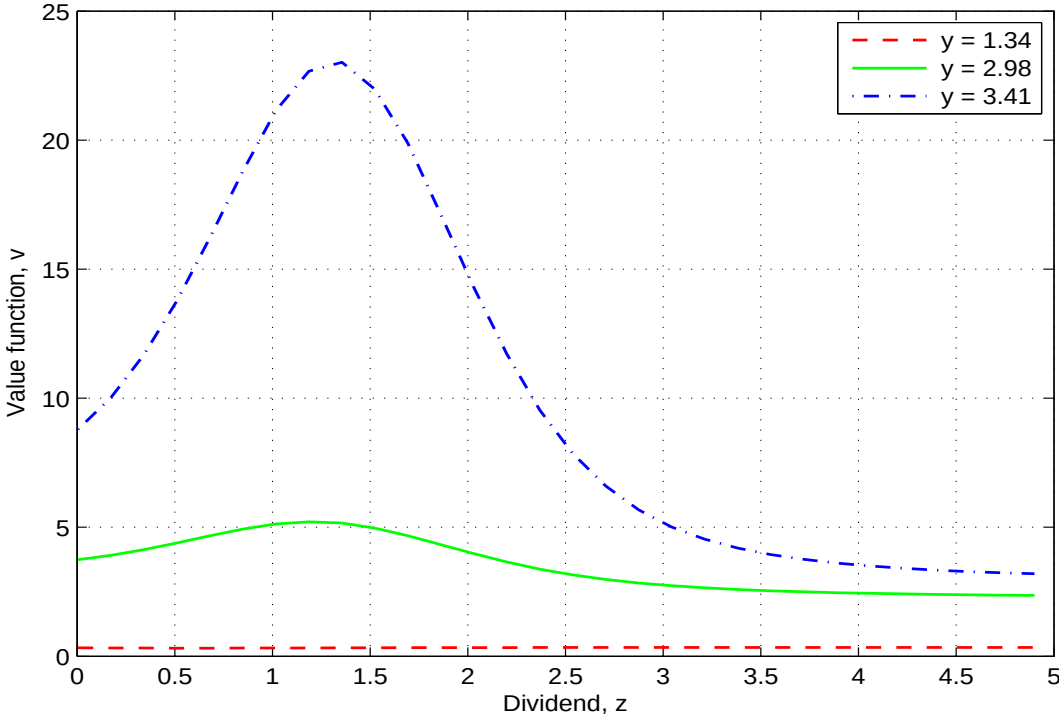



FIGURE 2. Linear variation of the value function $v$ over the dividend level $z$ for some specific values of profit level $y$ with $r = 0.2$, $\mu = 1.5$, $\alpha = 0.02$ and $\sigma = 1.3$.

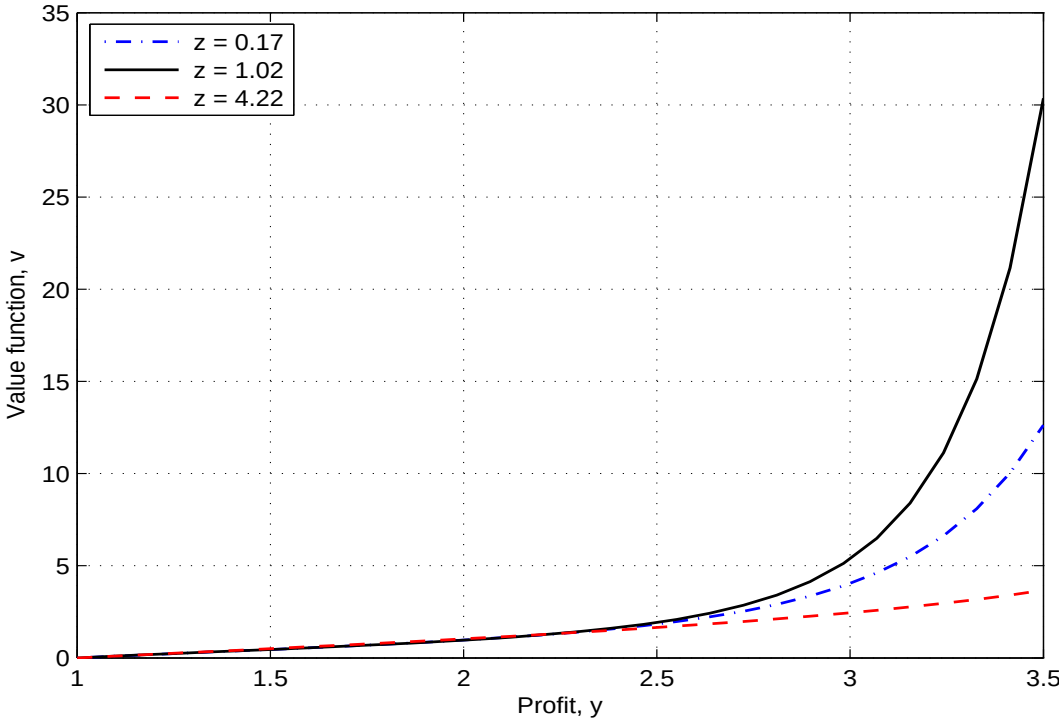



FIGURE 3. Linear variation of the value function $v$ over the profit level $y$ for some specific values of dividends $z$ with $r = 0.2$, $\mu = 1.5$, $\alpha = 0.02$ and $\sigma = 1.3$.

TABLE 1. The ratios of the optimal dividends over the profits made in relation to the highest possible profit

| Highest possible profit | Profit made | Optimal dividend | Optimal dividend/Profit |
|---|---|---|---|
| 3.50 | 2.98 | 1.19 | 0.40 |
| 3.60 | 3.06 | 1.19 | 0.39 |
| 3.70 | 3.14 | 1.19 | 0.38 |
| 3.80 | 3.22 | 1.19 | 0.37 |
| 3.90 | 3.30 | 1.36 | 0.41 |
| 4.00 | 3.38 | 1.36 | 0.40 |
| 4.10 | 3.46 | 1.36 | 0.39 |
| 4.20 | 3.54 | 1.36 | 0.38 |
| 4.30 | 3.62 | 1.36 | 0.37 |
| 4.40 | 3.70 | 1.36 | 0.37 |
| 4.50 | 3.78 | 1.36 | 0.36 |
| 4.60 | 3.86 | 1.52 | 0.40 |
| 4.70 | 3.93 | 1.52 | 0.39 |
| 4.80 | 4.01 | 1.52 | 0.38 |
| 4.90 | 4.09 | 1.52 | 0.37 |
| 5.00 | 4.17 | 1.52 | 0.37 |
| 5.10 | 4.25 | 1.52 | 0.36 |
| 5.20 | 4.33 | 1.69 | 0.39 |
| 5.30 | 4.41 | 1.69 | 0.38 |
| 5.40 | 4.49 | 1.69 | 0.38 |
| 5.50 | 4.57 | 1.69 | 0.37 |
| 5.60 | 4.65 | 1.69 | 0.36 |
| 5.70 | 4.73 | 1.69 | 0.36 |
| 5.80 | 4.81 | 1.86 | 0.39 |
| 5.90 | 4.89 | 1.86 | 0.38 |
| 6.00 | 4.97 | 1.86 | 0.37 |

Figure 4 shows how the impact of interest rate on investment eventually affects the value function on the dividend axis. Actually, this lowers the value function but the point of dividend that produces the highest value remains the same. Similarly, Figure 5 shows that the value function is lowered in the direction of the profit. In addition we learn that the impact is more significant for the higher values of the profit which are favourable for paying dividend.

The Figures 6 and 7 are about the situation when steadiness in dividend payment is applied without growth in the business. In Figure 6 we find out that the value function drops significantly if some portion of profit is not allocated for investment. This fact is also confirmed in Figure 7 from which we also learn that it is more observable when the profit is high. So, this means that having some portion of profit directed to the expansion of investment produces better results by adding more values to the shareholders.

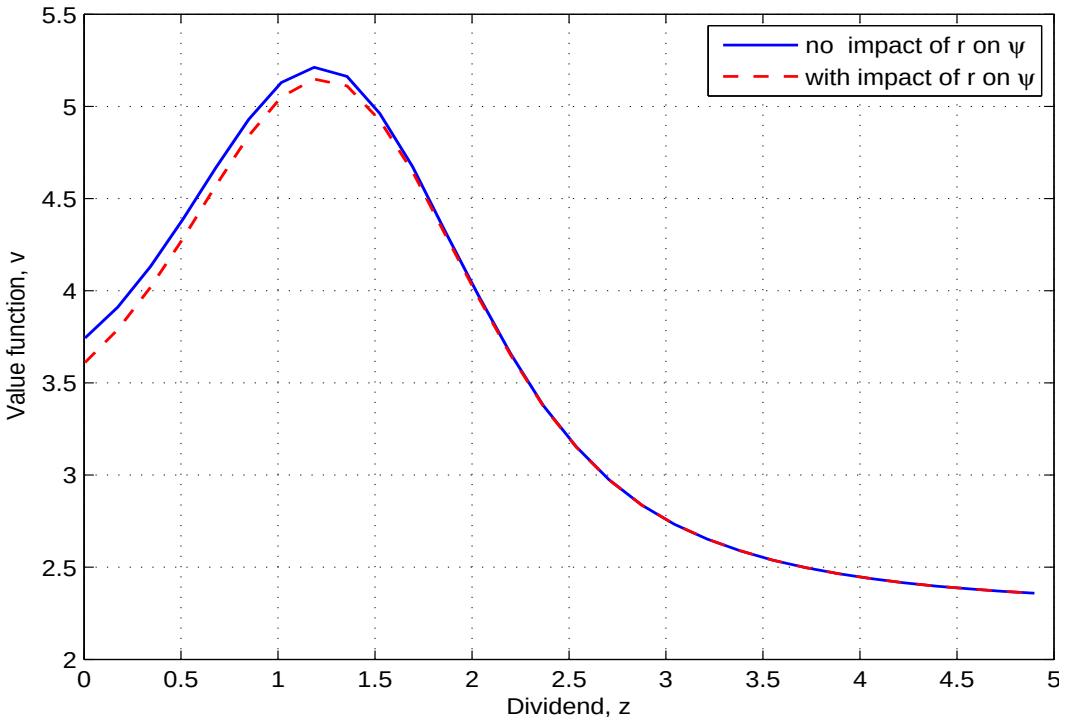



FIGURE 4. The impact of interest rate on investment over the variation of the value function $v$ and the dividend levels $z$ with $y = 2.98$, $r = 0.2$, $\mu = 1.5$, $\alpha = 0.02$ and $\sigma = 1.3$.

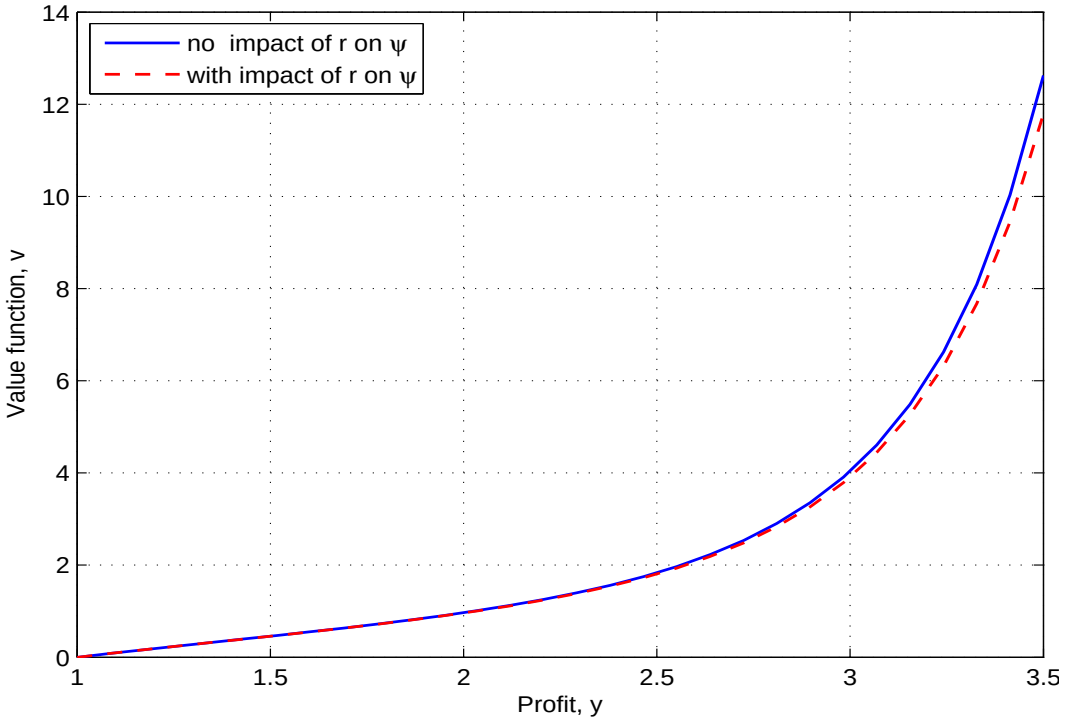



FIGURE 5. The impact of interest rate on investment over the variation of the value function $v$ and the profit levels $y$ with $z = 0.17$, $r = 0.2$, $\mu = 1.5$, $\alpha = 0.02$ and $\sigma = 1.3$.

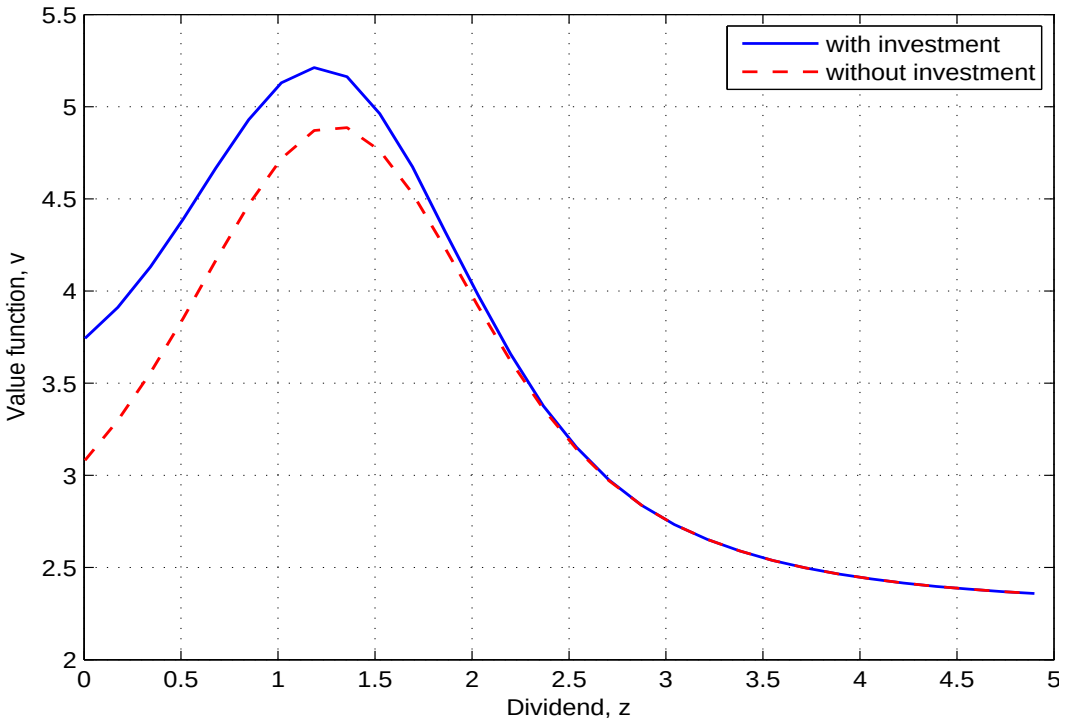


FIGURE 6. Comparing the value function $v$ when investment is applied and when it is not applied over the dividend levels $z$ with $y = 2.98$, $r = 0.2$, $\mu = 1.5$ and $\sigma = 1.3$.

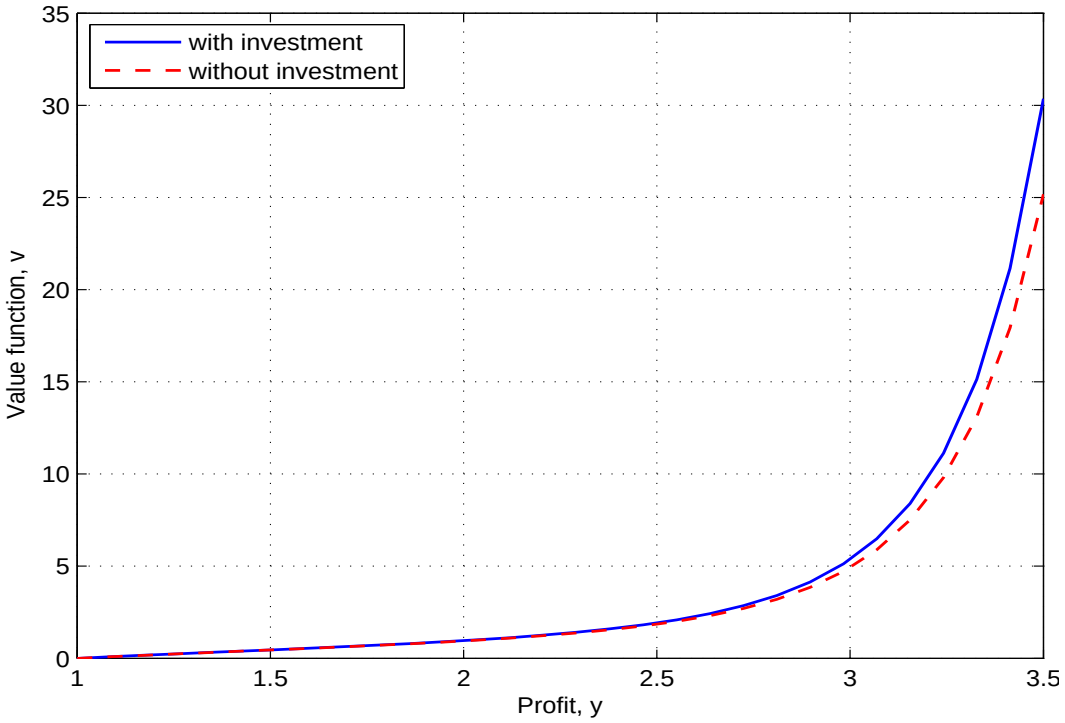



FIGURE 7. Comparing the value function $v$ when investment is applied and when it is not applied over the profit levels $y$ with $z = 1.02$, $r = 0.2$, $\mu = 1.5$ and $\sigma = 1.3$.

Figure 8 comprises of two plots. The first plot is about the variation of the value function over the dividends for both the exponential and the linear response of the investment function over investment amount as given in the equations (5) and (7). It is revealed by visual inspection that the two increasing functions give almost the same results, a fact which is substantiated in the second plot. The second plot is the histogram for the differences between the exponential and linear responses of the investment function. We see that the differences are negligible. Therefore, with some choice of parameters, any increasing function can serve as the representation of the investment function.

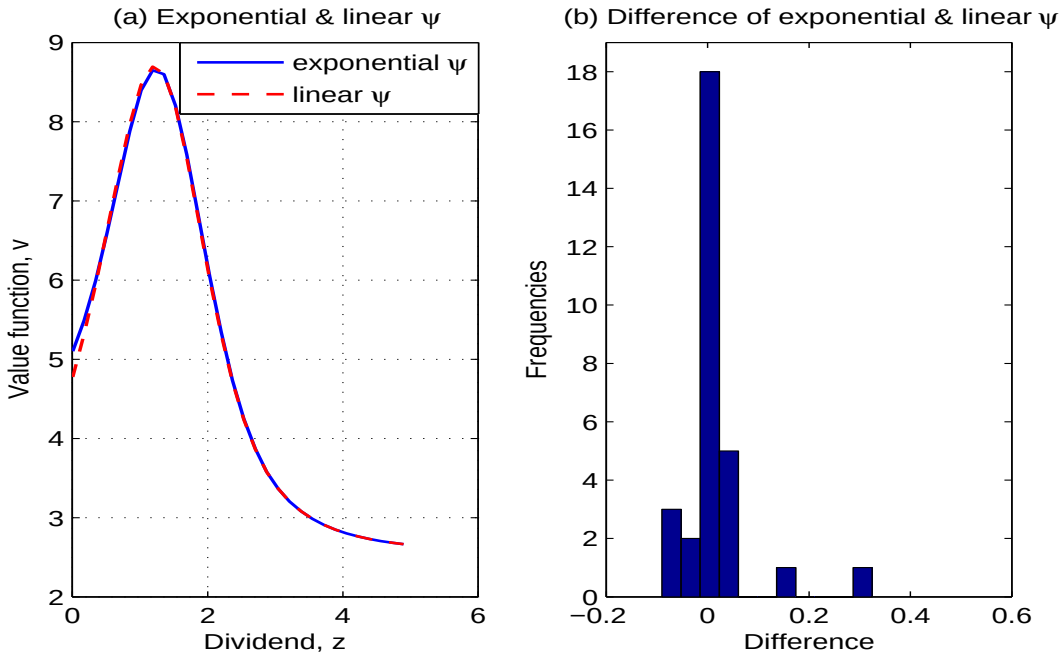



FIGURE 8. The difference between exponential and linear response of the investment function $\psi$ over the investment amount $I_t$ with the parameters $r = 0.2$, $\mu = 1.5$, $\alpha = 0.02$, $\sigma = 1.3$, $m = 0.06$ and $c = 0.01$.

## 5. Conclusion, Recommendation and Possible extensions

In this study we have proposed a strategy that can be adopted by firms' managers who want to satisfy their shareholders by steady and growing dividend payments meanwhile committing for investment expansion. The strategy is more useful for firms in developing economies where the demand for dividends is very high and most of firms need to be growing. We have considered companies whose dynamics of the profit follows a drifted Brownian motion.

We have been able to show the variation of the optimal value function over the profit level and the anticipated dividend amounts. In particular, we showed that the value function exponentially increases as the profit increases and it starts by increasing for the low values of dividend and eventually decreases over increasing value of dividend. This creates a localmaximum which gives the optimal dividend point. We therefore recommend that companies should use 36% of their profit as the benchmark for dividend in order to ensure steady dividend payment. The rest (i.e., 64%) should be retained for investment expansion and for the buffer that

will later on support dividend. Next we showed that the negative effect of the interest rate on the firms' investment ultimately lowers the optimal value function. Thus having low interest rates is favourable for the proposed approach and firm management in general. Then we compared a situation with steady dividend without investment to when investment expansion is considered and found that steady dividend with investment gives a better result. We also investigated the nature of the investment function, which is supposed to be increasing over investment amount, and found that it gives the same result for the exponential and linear formulation. Therefore, we recommend this strategy for companies in developing economies in order that they may satisfy the shareholders with steady and growing dividends while fulfilling their preference in expanding the investments by internal sources. Economic and financial policies in such economies have to ensure that the interest rates are as low as possible and establish regulations which can emphasize companies to pay dividends.

This study can be extended further by incorporating debt variable in the model and find the impact of debt financing on the investment. The same thinking can apply for equity financing. Also the stochastic interest rates can be applied as the discounting factor in the objective function formulation. All these are likely to improve the results and complicate the analysis but we reserve them for future considerations.

## Conflict of Interests

The authors declare that there is no conflict of interests.

## Acknowledgments

The study by Adeline P. Mtunya is under the sponsorship of the Government of the United Republic of Tanzania through the Commission for Science and Technology (COSTECH). The authors thank the sponsor for the financial grant that was necessary towards accomplishment of this work. We also appreciate the material and moral support from the management of The Nelson Mandela African Institution of Science and Technology (NM-AIST) and Mkwawa University College of Education (MUCE).

## REFERENCES


[1] F. Black, The dividend puzzle, J. Portfolio Manage. 2 (2) (1976), 5-8.

[2] M. Moradi, M. Salehi and S. Honarmand, Factors Affecting Dividend Policy: Empirical Evidence of Iran, Poslovna izvrsnos. Sveučilište u Zagrebu, Ekonomski fakultet Zagreb. 4 (1) (2010), 45-61.

[3] J. Bistrova and N. Lace, Dividend stability and sustainability in CEE region, Proceedings of the 2nd World Sustainability Forum. 2 (2012), 1-8.

[4] A. Brav, J. R. Graham, C. R. Harvey and R. Michaely, Payout policy in the 21st century, J. Financ. Econ. 77 (3) (2005), 483-527.

[5] M. Salehi, Relationship between Going Concern Concept and P/E Ratio in Emerging Market: Case of Iran, J. Manage. Res. 1 (1) (2008).

[6] H. DeAngelo, L. DeAngelo and R. Stulz, Dividend policy and the earned/contributed capital mix: a test of the life-cycle theory, J. Financ. Econ. 81 (2) (2006), 227-254.

[7] U. Uwuigbe, J. Jafaru and A. Ajayi, Dividend policy and firm performance: A study of listed firms in Nigeria, Accounting and Management Information Systems. Bucharest Acad. Econ. Stud. 11 (3) (2012), 442.

[8] S. C. Myers and N. S. Majluf, Corporate financing and investment decisions when firms have information that investors do not have, J. Financ. Econ. 13 (2) (1984), 187-221.

[9] S. Agarwal and H. Mohtadi, Financial markets and the financing choice of firms: evidence from developing countries, Glob. Financ. J. 15 (1) (2004), 57-70.

[10] A. P. Mtunya, P. Ngare and Y. Nkansah-Gyekye, On Steady Dividend Payment under Functional Mean Reversion Speed, J. Math. Financ. 6 (3) (2016), 368.

[11] J. Lintner, Distribution of incomes of corporations among dividends, retained earnings, and taxes, Am. Econ. Rev. 46 (2) (1956), 97-113.

[12] M. Miller and F. Modigliani, Dividend policy, growth, and the valuation of shares, J. Bus. 34 (4) (1961), 411-433.

[13] M. J. Gordon, Optimal investment and financing policy, J. Financ. 18 (2) (1963), 264-272.

[14] M. Jeanblanc-Picqué and A. N. Shiryaev, Optimization of the flow of dividends, Russ. Math. Surv. 50 (2) (1995), 257.

[15] G. E. Farrelly, H. K. Baker and R. B. Edelman, Corporate Dividends-views of the Policy-makers, Akron Bus. Econ. Rev. 17 (4) (1986), 62-74.

[16] R. D. Arnott, Dividends and the three dwarfs, Financ. Anal. J. 59 (2) (2003), 4-6.

[17] S. Gul, M. Sajid, N. Razzaq, M. Iqbal and M. B. Khan, The relationship between dividend policy and shareholders wealth, Econ. Financ. Rev. 2 (2) (2012), 55-59.

[18] R. Radner and L. Shepp, Risk vs. profit potential: A model for corporate strategy, J. Econ. Dyn. Control 20 (8) (1996), 1373-1393.

[19] J. Décamps and S. Villeneuve, Optimal dividend policy and growth option, Finance Stoch. 11 (1) (2007), 3-27.

[20] X. Wang and Y. Wang, Optimal investment and consumption with stochastic dividends, Appl. Stoch. Models Bus. Ind. 26 (6) (2010), 792-808.

[21] E. Chevalier, V. L. Vath and S. Scotti, An optimal dividend and investment control problem under debt constraints, SIAM J. Financ. Math. 4 (1) (2013), 297-326.

[22] X. Han and F. Yi,A stochastic control model of investment, production, and consumption on a finite horizon, Math. Methods Appl. Sci. 38 (6) (2015), 1070-1080.

[23] Z. Jiang and M. Pistorius, Optimal dividend distribution under Markov regime switching, Financ. Stoch. 16 (3) (2012), 449-476.

[24] Z. Jin, G. Yin and C. Zhu, Numerical solutions of optimal risk control and dividend optimization policies under a generalized singular control formulation, Automatica. 48 (8) (2012), 1489-1501.

[25] E. Chevalier, M. Gaïgi and V. L. Vath, Liquidity risk and optimal dividend/investment strategies, Math. Financ. Econ. 11 (1) (2017), 111-135.

[26] C. Adaoglu, Instability in the dividend policy of the Istanbul Stock Exchange (ISE) corporations: evidence from an emerging market, Emerg. Mark. Rev. 1 (3) (2000), 252-270.

[27] Q. T. Tran and T. T. H. Nguyen, Dividend Policy Behavior in Emerging Stock Markets: Evidence from Vietnamese Stock Market, Int. J. Financ. Res. 5 (4) (2014), 85.

[28] L. V. Dewri and M. R. Islam, Behavioral Analysis of Investors' Attitude towards Dividend Declaration in Developing Country-A Case of Bangladesh, Int. J. Bus. Manage. 10 (11) (2015), 185.

[29] A. Løkka and M. Zervos,Optimal dividend and issuance of equity policies in the presence of proportional costs, Insur. Math. Econ. 42 (3) (2008), 954-961.

[30] E. Akyildirim, I. E. Güney, J. Rochet, and H. M. Soner, Optimal dividend policy with random interest rates, J. Math. Econ. 51 (2014), 93-101.

[31] E. Pierre, S. Villeneuve and X. Warin, Capital Investment and Liquidity Management with collateralized debt, TSE Working Paper. Toulouse 1 Capitole Publications. (2014), 14-542.