UPDATING STRUCTURAL PARAMETERS OF A FINITE ELEMENT MODEL BY ADDING MASS OR STIFFNESS TO THE SYSTEM

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ABSTRACT

Whilst the techniques of structural dynamic modelling are well established it is difficult to produce accurate dynamic models of complex structures and for critical applications model updating using experimental data is necessary. A number of methods have been proposed in the literature but most of these have difficulties in handling the problem of incomplete modes and unmeasurable degrees of freedom. In this paper a method is presented which is based on exact model reduction and perturbation of both the actual structure and its analytical model by adding mass or stiffness. Eigendata sensitivities for the unperturbed and the perturbed systems generate a set of linearly independent equations for which a unique solution exists from incomplete modes. With measurement errors, a statistical Bayesian estimator yields an improved model which can better predict responses of unmeasured DOF over a larger frequency range.

NOMENCLATURE

$\mathbf{K}, \mathbf{M}, \mathbf{D}$	Stiffness, mass, dynamic stiffness matrices.
$\mathbf{K}_{11}, \mathbf{M}_{11}, \mathbf{D}_{11}$	Submatrices corresponding to retained DOF.
$\mathbf{K}_{22}, \mathbf{M}_{22}, \mathbf{D}_{22}$	Submatrices corresponding to reduced DOF.
$\mathbf{K}_{12}, \mathbf{K}_{21}$, *etc*	Submatrices coupling retained and reduced degrees of freedom.
$\delta\mathbf{M}, \delta\mathbf{K}$	Mass and stiffness addition matrices.
$\mathbf{F}$	Forcing vector.
$\mathbf{Y}$	Mass normalized modeshape vector
$\mathbf{Y}_1, \mathbf{Y}_2$	Subvectors corresponding to retained and reduced degrees of freedom respectively.
$\mathbf{U}$	Eigenvector of the reduced order model.
λ $(= \omega^2)$	Eigenvalue (equal to natural freq. squared).
$\mathbf{J}_\lambda$	Jacobian matrix of eigenvalue sensitivities.
$\mathbf{J}_y$	Jacobian matrix of eigenvector sensitivities.
$\mathbf{W}$	Inverse covariance matrix of measured data.
$\mathbf{W}_o$	Inverse covariance matrix of initial parameter estimates.
$\mathbf{s}$	Structural parameter vector.
$\Delta\mathbf{s}$	Difference between correct and current parameter estimates.
$\Delta\mathbf{s}_o$	Difference between initial and current parameter estimates.
$\Delta\lambda$	Difference between measured and analytical eigenvalues.
$\Delta\mathbf{Y}, \Delta\mathbf{U}$	Difference between measured and analytical eigenvectors.
N	Number of model degrees of freedom.
n	Number of degrees of freedom of the reduced order model.
$\mathbf{R}$	An arbitrary matrix.
$\mathbf{A}, \mathbf{L}$	Eigenvector and eigenvalue matrices.
$\mathbf{B}$	Matrix of selected vectors.
q	Number of coordinates with additional mass or stiffness.
p	Number of measured modes.
U, T	Strain energy, Kinetic energy.
a	Constants of linear dependence.
m	Mass per unit length of a beam.
EI	Flexural rigidity of a beam.

Superscript

*	Perturbed system.
T	Matrix transpose.

Subscript

r, i, t, g	rth, ith, tth and gth mode.
m, k	Mass addition , stiffness addition.
x, R	Arbitrary model, reduced order model.

INTRODUCTION

Methods of dynamic modelling and analysis of structures, for example the finite element method, are well developed. Unfortunately theoretical analysis alone is not adequate to produce an accurate dynamic model of a complex structure and for critical applications, model updating using experimental data measured on real structures is necessary.

A number of techniques for updating dynamic models using vibration test data have been proposed[1-12] but no technique has received a general acceptance due to a number of difficulties. The two main difficulties are (i) the incomplete modal information from vibration tests and (ii) the inability to make measurements at all degrees of freedom corresponding to the analytical model. When incomplete modal data is used, there is an infinite number of model parameters that could reproduce the measured data.[1] This problem has been dealt with by using statistical methods which involves a constraint

of minimum parameter changes from the initial estimates.[1-4] Although this constraint preserves the physical significance of the updated parameters, a more accurate model could be found if the modal data had a unique solution for the parameters.

Theoretically the dynamic response of a system contains all modes of the system. In practice, if response data is used it is found that the contribution of modes far away from the excitation frequencies is very small and completely overshadowed by measurement noise. Thus frequency domain techniques which use the equation error formulation experience severe bias problems since the polluted response data is contained in the coefficient matrix. Fritzen[5] proposes to reduce bias by using an instrumental variable method. Natke *et al*[6-8] avoids bias by using a non-linear frequency domain output error formulation. Neither Natke nor other investigators have successfully addressed the problem of mismatch between the number of model DOF and measurement DOF. Computation of the unmeasured DOF[3,13] and reduction of the analytical model are two proposed methods. However, both are inexact and therefore liable to errors. Reduction procedures used are approximate ones which ignore the frequency dependence of the reduced spatial matrices.

In this paper a technique is presented which attempts to overcome both the problem of incomplete modes and the mismatch between the number of model DOF and measurement DOF. It is based on exact model reduction and sensitivities of the eigendata of the analytical model before and after the structure and its analytical model are equally perturbed by adding mass or stiffeners.

THEORETICAL DEVELOPMENT

UPDATING USING EIGENVALUES AND EIGENVECTORS

Consider a dynamic system which is adequatelly represented by an Nth order model of symmetric mass and stiffness matrices. Parameter changes $\Delta\mathbf{s}$ required for the analytical model to reproduce measured eigendata can be approximately written as a function of measured data using sensitivity analysis as:

$$\begin{bmatrix}\mathbf{J}_\lambda\\ \mathbf{J}_y\end{bmatrix}\Delta s = \begin{Bmatrix}\Delta\lambda\\ \Delta\mathbf{Y}\end{Bmatrix} \tag{1}$$

With p incomplete modes ($p < N$), (1) has an infinite number of solutions, even if it is overdetermined. If the system is now perturbed by adding mass or stiffness, the equation of motion for the ith mode becomes:

$$[(\mathbf{K} + \delta\mathbf{K}) - \lambda_i^*(\mathbf{M} + \delta\mathbf{M})]\mathbf{Y}_i^* = 0 \tag{2a}$$

or

$$[\mathbf{K} - \lambda_i^*\mathbf{M}]\mathbf{Y}_i^* = \mathbf{F}(\lambda_i^*) \tag{2b}$$

where $\mathbf{F}(\lambda_i^*) = [\lambda_i^*\delta\mathbf{M} - \delta\mathbf{K}]\mathbf{Y}_i^*$ is an equivalent forcing vector.

Equation (2b) transforms the modal data of the perturbed system into forced response data of the original unperturbed system, valid only at frequencies corresponding to the natural frequencies of the perturbed system. It follows therefore the incomplete modal data of the perturbed system contains all modal information of the original unperturbed system and may be used to find the unique model parameters of the original system.

The approach described in this paper is to add identical masses or stiffeners to both the analytical model and the actual structure (so as to keep $\Delta\mathbf{s}$ unchanged) and measure the incomplete modal data. This is repeated at a number of coordinates using the same or different masses or stiffeners so that (1) is overdetermined. With error free data, exact parameters will be obtained. However because of measurement errors a suitable statistical technique must be employed. In this paper a Bayesian estimator is used incorporating a constraint of minimum changes on the initial estimates.

$$\Delta\mathbf{s} = \left(\begin{bmatrix}\mathbf{J}_\lambda\\ \mathbf{J}_y\end{bmatrix}^T[\mathbf{W}]\begin{bmatrix}\mathbf{J}_\lambda\\ \mathbf{J}_y\end{bmatrix} + [\mathbf{W}_o]\right)^{-1}\left(\begin{bmatrix}\mathbf{J}_\lambda\\ \mathbf{J}_y\end{bmatrix}^T[\mathbf{W}]\begin{Bmatrix}\Delta\lambda\\ \Delta\mathbf{Y}\end{Bmatrix} + [\mathbf{W}_o]^T\{\Delta\mathbf{s}_o\}\right) \tag{3}$$

The simplest perturbation is to add a lumped mass or grounded stiffener. If the number of measurement DOF is smaller than the number of DOF of the analytical model, the model has to be reduced to the measurement DOF and appropriate sensitivities for the reduced order model, which are derived elsewhere in the paper, used in (3).

UPDATING USING EIGENVALUES ALONE

The eigenvalues are solutions of the determinant equations:

$$\det[\mathbf{K} - \lambda\mathbf{M}] = 0, \quad \det[\mathbf{K} + \delta\mathbf{K} - \lambda^*(\mathbf{M} + \delta\mathbf{M})] = 0 \tag{4}$$

Let $\mathbf{R}$ be a non-singular but otherwise arbitrary N th order matrix such that $\mathbf{R}\,\delta\mathbf{K}\mathbf{R}^T = \delta\mathbf{K}$ and $\mathbf{R}\,\delta\mathbf{M}\mathbf{R}^T = \delta\mathbf{M}$. Then

$$\det[\mathbf{R}\mathbf{K}\mathbf{R}^T - \lambda\mathbf{R}\mathbf{M}\mathbf{R}^T] = 0 \tag{5a}$$
$$\det[\mathbf{R}\mathbf{K}\mathbf{R}^T + \delta\mathbf{K} - \lambda^*(\mathbf{R}\mathbf{M}\mathbf{R}^T + \delta\mathbf{M})] = 0 \tag{5b}$$

Therefore $\mathbf{K}_x = \mathbf{R}\mathbf{K}\mathbf{R}^T$ and $\mathbf{M}_x = \mathbf{R}\mathbf{M}\mathbf{R}^T$ are arbitrary stiffness and mass matrices that could reproduce measured eigenvalues. The objective is to ensure that $\mathbf{K}_x$ and $\mathbf{M}_x$ are unique. Now $\delta\mathbf{K}$ and $\delta\mathbf{M}$ are symmetric and can be written as:

$$\delta\mathbf{K} = \mathbf{A}_k\mathbf{L}_k\mathbf{A}_k^T, \quad \delta\mathbf{M} = \mathbf{A}_m\mathbf{L}_m\mathbf{A}_m^T \tag{6}$$

where $\mathbf{A}_k$, $\mathbf{L}_k$ and $\mathbf{A}_m$, $\mathbf{L}_m$ are eigenvector and eigenvalue matrices of $\delta\mathbf{K}$ and $\delta\mathbf{M}$.

Consider a simple case of mass addition only ($\delta\mathbf{K} = \mathbf{0}$) at a single coordinate. Then $\delta\mathbf{M}$ has one nonzero eigenvalue and one unique eigenvector. But using (6),

$$\delta\mathbf{M} = \mathbf{R}\,\delta\mathbf{M}\,\mathbf{R}^T = \mathbf{R}(\mathbf{A}_m\mathbf{L}_m\mathbf{A}_m^T)\mathbf{R}^T = (\mathbf{R}\mathbf{A}_m)\mathbf{L}_m(\mathbf{R}\mathbf{A}_m)^T \tag{7}$$

Therefore $\mathbf{R}\mathbf{A}_m$ is also an eigenvector matrix of $\delta\mathbf{M}$. The eigenvector matrices $\mathbf{R}\mathbf{A}_m$ and $\mathbf{A}_m$ have one identical vector. If mass addition is now repeated one at a time at q coordinates ($q < N$) there will be q eigenvector matrices $\mathbf{A}_m$ where each set of the matrices $\mathbf{A}_m$ and $\mathbf{R}\mathbf{A}_m$ have one unique and identical vector corresponding to mass addition to one coordinate. For the q pair of identical vectors

$$\mathbf{RB} = \mathbf{B} \tag{8}$$

Where **B** is an N xq matrix of the q unique vectors. Equation (8) represents N x q equations in N^2 unknowns. Therefore **R** is non unique. If mass is added at least to every coordinate then there are N identical vectors and **R** is an identity matrix according to (9).

$$\mathbf{RB} = \mathbf{B} \tag{9}$$

Therefore it is possible to find exact parameters of a model using eigenvalues alone of the unperturbed and the perturbed systems provided mass is added at least once to every coordinate and (1) based on eigenvalue sensitivities alone is not underdetermined.

DYNAMIC MODEL REDUCTION

Consider the equation of motion

$$(\mathbf{K} - \lambda\mathbf{M})\mathbf{Y} = \mathbf{0}$$

Model reduction is performed by partitioning this equation of motion into submatrices corresponding to measured and unmeasured DOF. Thus, the reduced modal equation of motion can be shown[14] to be :

$$(\mathbf{D}_{11} - \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\mathbf{D}_{21})\mathbf{Y}_1 = \mathbf{D}_R\mathbf{Y}_1 = \mathbf{0} \tag{10}$$

and

$$\mathbf{Y}_2 = -\mathbf{D}_{22}^{-1}\mathbf{D}_{21}\mathbf{Y}_1 \tag{11}$$

where

$$\mathbf{D}_{12} = \mathbf{K}_{12} - \lambda\mathbf{M}_{12},\ \mathbf{D}_{21} = \mathbf{K}_{21} - \lambda\mathbf{M}_{21},$$
$$\mathbf{D}_{22} = \mathbf{K}_{22} - \lambda\mathbf{M}_{22}$$

The partitioning leads to an equation of motion nonlinear in λ. Previous investigators have used an approximate reduced equation which ignore this nonlinear dependence on the eigenvalue. This is only approximately correct when the kinetic energy associated with the deleted DOF is negligible. In a vibration test some of the coordinates that cannot be measured may have significant kinetic energy. In this paper exact expressions for the reduced mass and stiffness matrices are derived. These are then used with (10) to find sensitivities of the eigendata. The reduced mass and stiffness matrices are found using energy relationships thus:

$$U = \frac{1}{2}\left\{\mathbf{Y}_1^T\ \mathbf{Y}_2^T\right\}\begin{bmatrix}\mathbf{K}_{11} & \mathbf{K}_{12}\\ \mathbf{K}_{21} & \mathbf{K}_{22}\end{bmatrix}\begin{Bmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{Bmatrix} = \frac{1}{2}\mathbf{Y}_1^T\mathbf{K}_R\mathbf{Y}_1 \tag{12a}$$

$$T = \frac{1}{2}\left\{\dot{\mathbf{Y}}_1^T\ \dot{\mathbf{Y}}_2^T\right\}\begin{bmatrix}\mathbf{M}_{11} & \mathbf{M}_{12}\\ \mathbf{M}_{21} & \mathbf{M}_{22}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{Y}}_1\\ \dot{\mathbf{Y}}_2\end{Bmatrix} = \frac{1}{2}\dot{\mathbf{Y}}_1^T\mathbf{M}_R\dot{\mathbf{Y}}_1 \tag{12b}$$

Substitute (11) into (12) and after simplification, it can be shown that the reduced mass and stiffness matrices are given by:

$$\mathbf{M}_R = \mathbf{M}_{11} - \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\mathbf{M}_{21} - \mathbf{M}_{12}\mathbf{D}_{22}^{-1}\mathbf{D}_{21} + \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\mathbf{M}_{22}\mathbf{D}_{22}^{-1}\mathbf{D}_{21} \tag{13a}$$

$$\mathbf{K}_R = \mathbf{K}_{11} - \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\mathbf{K}_{21} - \mathbf{K}_{12}\mathbf{D}_{22}^{-1}\mathbf{D}_{21} + \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\mathbf{K}_{22}\mathbf{D}_{22}^{-1}\mathbf{D}_{21} \tag{13b}$$

Where the eigenvalues used in the computation of $\mathbf{M}_R$ and $\mathbf{K}_R$ are obtained using **K** and **M** by standard techniques for the eigensolution of nonfrequency dependent matrices.

SENSITIVITIES OF THE EIGENVALUE AND EIGENVECTOR OF THE REDUCED ORDER MODEL

The derivation of eigenvalue and eigenvector sensitivities for symmetric frequency independent matrices is well documented in the literature. The approach used by Fox and Kapoor[15] to derive eigenvalue and eigenvector sensitivities is extended here for frequency dependent mass and stiffness matrices.

Let **U** denote orthonormal eigenvectors of the reduced order model. Then only one of these vectors, say $\mathbf{U}_r$, will be identical to the subvector $\mathbf{Y}_1$ of the full order model. Thus premultiply (10) by $\mathbf{U}_r^T$, where $\mathbf{Y}_1$ is replaced by $\mathbf{U}_r$, and differentiate with respect to structural parameters leads to:

$$\frac{d\mathbf{U}_r^T}{ds}\mathbf{D}_R\mathbf{U}_r + \mathbf{U}_r^T\left[\frac{\partial\mathbf{D}_R^T}{\partial s} + \frac{\partial\mathbf{D}_R}{\partial\lambda}\frac{d\lambda}{ds}\right]\mathbf{U}_r + \mathbf{U}_r^T\mathbf{D}_R\frac{d\mathbf{U}_r}{ds} = 0 \tag{14}$$

The first and last term in (14) are zero. Substitute for $\mathbf{D}_R$ into (14) and after rearrangement the eigenvalue sensitivity is:

$$\frac{d\lambda}{ds} = \mathbf{U}_r^T\left[\frac{\partial\mathbf{D}_{11}}{\partial s} - \frac{\partial\mathbf{D}_{12}}{\partial s}\mathbf{D}_{22}^{-1}\mathbf{D}_{21} - \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\frac{\partial\mathbf{D}_{21}}{\partial s} + \mathbf{D}_{12}\mathbf{D}_{22}^{-1}\frac{\partial\mathbf{D}_{22}}{\partial s}\mathbf{D}_{22}^{-1}\mathbf{D}_{21}\right]\mathbf{U}_r \tag{15}$$

To derive eigenvector sensitivity we consider a vector of the reduced model to be a linear combination of the eigenvectors of the reduced model evaluated at that frequency. The differential of an eigenvector is thus written as:

$$\frac{d\mathbf{U}_r}{ds} = \sum_{t=1}^{n} a_{r,t}\mathbf{U}_t \tag{16}$$

Differentiating (10) and using (16) we have

$$\mathbf{D}_R\sum_{t=1}^{n} a_{r,t}\mathbf{U}_t = -\left[\frac{\partial\mathbf{D}_R}{\partial s} + \frac{\partial\mathbf{D}_R}{\partial\lambda}\frac{d\lambda}{ds}\right]\mathbf{U}_r \tag{17}$$

Premultiply (17) by $\mathbf{U}_g^T$ $(g \neq r)$ and after simplification, For $g \neq r$:

$$a_{r,g} = \frac{\mathbf{U}_g^T\left[\frac{\partial\mathbf{D}_R}{\partial s} + \frac{\partial\mathbf{D}_R}{\partial\lambda}\frac{d\lambda}{ds}\right]\mathbf{U}_r}{\lambda_r - \lambda_g} \tag{18}$$

For $g = r$, we differentiate the mass orthogonality relationship for the mass normalized modes and express the eigenvector differential using (16) to arrive to the following relationship:

$$a_{r,r} = -\frac{1}{2}\mathbf{U}_r^T\left[\frac{\partial \mathbf{M}_R}{\partial \mathbf{s}} + \frac{\partial \mathbf{M}_R}{\partial \lambda}\frac{d\lambda}{d\mathbf{s}}\right]\mathbf{U}_r \qquad (19)$$

Using (16), (17), (18) & (19) the eigenvector sensitivity becomes:

$$\frac{d\mathbf{U}_r}{d\mathbf{s}} = \sum_{\substack{g=1 \\ g\neq r}}^{n}\left\{\frac{\mathbf{U}_g}{\lambda_r - \lambda_g}\mathbf{U}_g^T\left[\frac{\partial \mathbf{D}_R}{\partial \mathbf{s}} + \frac{\partial \mathbf{D}_R}{\partial \lambda}\frac{d\lambda}{d\mathbf{s}}\right]\mathbf{U}_r\right\} -$$

$$\frac{1}{2}\mathbf{U}_r\mathbf{U}_r^T\left[\frac{\partial \mathbf{M}_R}{\partial \mathbf{s}} + \frac{\partial \mathbf{M}_R}{\partial \lambda}\frac{d\lambda}{d\mathbf{s}}\right]\mathbf{U}_r \qquad (20)$$

NUMERICAL EXAMPLE

A plane frame shown in Fig 1 consists of six beam members and is modelled by a FE model comprising ten beam elements with a total of 24 DOF to simulate a free test condition. Each of the nodes n1 - n8 has two translational and one rotational DOF in the x-z plane. Parameters to be updated are the mass per unit length m and flexural rigidity EI for each beam member. Correct parameters and initial estimates for the analytical model are given in Table 1 where values in brackets are the assumed standard deviations. It is also assumed that only the first three elastic modes are measured at only 4 coordinates. These are the transverse displacement at nodes n2 and n5 and the z-coordinates at nodes n7 and n8. The analytical model must therefore be dynamically reduced from 24 DOF to 4 DOF. The true natural frequencies and mass normalized modeshape vectors for the system are given below.

	Mode 1	Mode 2	Mode 3
	34.98Hz	62.24Hz	83.56Hz
n2	0.0962	0.2914	0.3041
n5	-0.0962	0.2914	-0.3041
n7	-0.1550	-0.0837	0.1177
n8	0.1550	-0.0837	-0.1177

Three model updating cases are examined. For each mass or stiffener addition only the first 3 elastic modes are used.

Case1.1 By adding mass at nodes n2 and n5 and using both the eigenvalues and eigenvectors. The added masses were 0.35Kg and 0.5Kg.

Case1.2 By adding mass at all four coordinates of the reduced order model (nodes n2 n5 n7 n8) and using eigenvalues only. The added masses were the same as in Case1.1.

Case1.3 By fixing nodes n1 and n6 to simulate a clamped clamped condition and adding grounded stiffeners at nodes n7 and n8. The stiffeners are 10^6N/m and 7.5×10^6N/m at n7, 2.5×10^6N/m and 5.0×10^6N/m at n8. Both eigenvalues and eigenvectors in the updating algorithm.

With error free eigendata an ordinary least squares method gave exact parameters in all cases. To simulate a noisy experimental data, random errors with a standard deviation of 0.5Hz was added to all the natural frequencies and a standard deviation of 0.02 was added to all modal displacements. Parameter updates obtained after four iterations, using the Bayesian estimator are given in Table 2. Further iterations did not have significant changes in the parameters. Fig 2 compares receptance predictions in the x direction at node n4 for a force input at node n8 for Case1.1 if the frame is now clamped at n1 and n6. Fig 3 compares receptance predictions in the z direction at node n3 for a force input at node n7 for Case1.2 if the frame is clamped at n1 and n6.

LABORATORY EXPERIMENT ON A CANTILEVER BEAM

An aluminium cantilever beam, 50x25x710mm long, is modelled by FEA using 7 beam elements with discrete translational and rotational stiffeners for support flexibility at the clamped end and a concentrated mass at node n5 for the mass loading of the shaker-beam interface as shown in Fig 4. Parameters to be updated are mass per unit length m, flexural rigidity EI, translational and rotational stiffeners at the clamped end and mass loading of the shacker-beam interface. Beam parameters are updated individually giving rise to a total of 17 unknowns. Initial estimates and standard deviations (in brackets) are given below.

EI = 4557Nm^2 (150Nm^2), m = 3.4Kg (0.1Kg) for all elements,
K_{trans} = 8.0×10^7N/m (2.0×10^7N/m),
K_{rot} = 4.0×10^5N/m (1.0×10^5N/m), m_{load} = 0 (0.1kg).

Only the first two modes were measured. Measured natural frequencies were 38.7Hz and 238Hz. Natural frequencies calculated by the initial model were 39.4Hz and 245.5Hz. Two updating studies have been undertaken, thus.

Case 2.1. Using eigenvalues only and by adding mass at 4 coordinates (nodes n3, n4, n6, n7). The model was dynamically reduced from 16DOF to 4DOF. The added masses were 0.199kg and 0.355kg.

Case 2.2. Using eigenvalues and eigenvectors with 7 measurement coordinates (nodes n2-n8) and mass added at 4 coordinates as in Case2.1.

From the experimental data a standard deviation of 0.1Hz and 0.2Hz was established for the first and second modes respectively and a standard deviation of 0.05 used for all modal displacements. The updated parameters after four iterations, are given in Table 3. Figs 5 and 6 compares receptance predictions at the excitation coordinate (node5) for Case 2.1 and at the tip of the beam for Case 2.2.

DISCUSSION

With error free data, the mass or stiffness addition technique presented gives exact parameters using an ordinary least squares solution. However with errors in the measured data, the weighted least squares method will not always give accurate parameters although the incomplete modes of the perturbed system contains complete modal information of the unperturbed system. This is because the weighted least squares estimator is not a minimum variance estimator and therefore parameters of which the eigendata is least sensitive to will be poorly estimated. The Bayesian estimator used in the paper is a simple one and helps to minimize the variance of such parameters. It may be possible to improve further the accuracy by using a constraint of minimum parameter changes on the current estimates rather than the initial estimates. However as subsequent parameter updates are correlated with measurement vector, for minimum variance, the weighting

matrices have to be updated at each iteration. A minimum variance algorithm with updated weighting matrices is given by Friswell[4].

CONCLUSION

A technique has been presented which attempts to overcome the problems of incomplete modes and mismatch between model DOF and measurement DOF. It is based on exact model reduction and sensitivities of the eigenvectors and eigenvalues or eigenvalues alone when the structure and its analytical model are equally perturbed by adding masses or stiffeners. The feasibility of the technique has been demonstrated on a simulated example and a laboratory experiment on a simple cantilever beam with support flexibility. While the mass addition technique can be successfully used for small structures eg turbine blades, the limitation of adding mass will be apparent for very large structures as the size of the mass to produce even small changes in eigendata may be objectionable. For such structures it may be more appropriate to add stiffeners. However, the practicality of doing this requires investigation.

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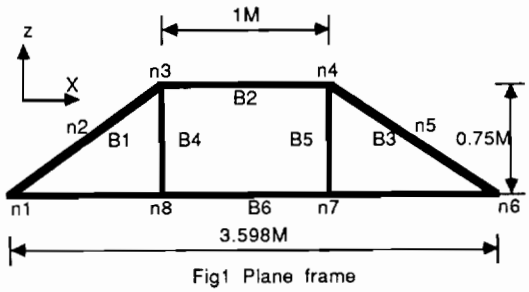


Fig1 Plane frame

		B1	B2	B3	B4	B5	B6
Correct parameters	EI(NM^2)	21333	9000	21333	5208	5208	14292
	m(Kg/M)	6.28	4.71	6.28	3.93	3.93	5.5
	EA(N) (*E+08)	1.6	1.2	1.6	1.0	1.0	1.4
Initial estimates	EI(NM^2) (STD)	25000 (5000)	10000 (2500)	19000 (5000)	4800 (500)	4900 (500)	18000 (2500)
	m(Kg/M) (STD)	6.0 (0.5)	4.0 (0.5)	5.5 (0.5)	4.5 (0.5)	4.5 (0.5)	6.0 (0.5)

Table1 Correct parameters and initial estimates
(Plane frame example)

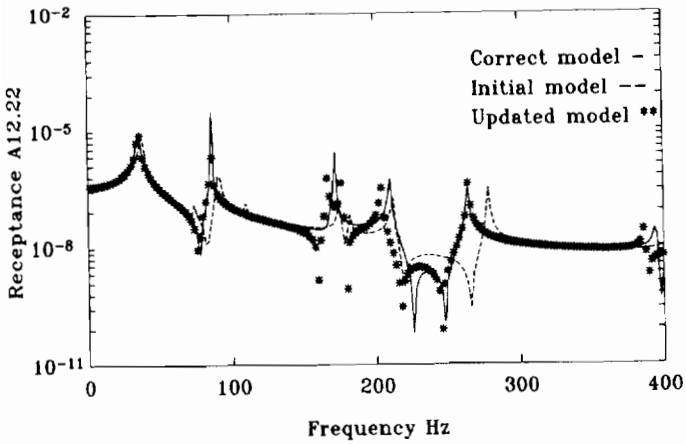


Fig 2. Plane frame receptance in the x-DOF at n4
for a force input in the z-DOF at n8.

Beam No	Updated Parameters Case 1.1 EI (NM^2)	Case 1.1 m (Kg/M)	Case 1.2 EI (NM^2)	Case 1.2 m (Kg/M)	Case 1.3 EI (NM^2)	Case 1.3 m (kg/M)
B1	19586	6.219	22127	6.305	21014	6.127
B2	10721	3.669	9463	4.133	8819	3.904
B3	19613	6.053	18242	5.841	19552	5.756
B4	4712	4.027	4720	4.543	4906	4.599
B5	5060	4.496	4777	4.491	4549	4.414
B6	14974	6.159	16641	6.034	15680	5.824

Table 2 Updated parameters = Plane Frame example.

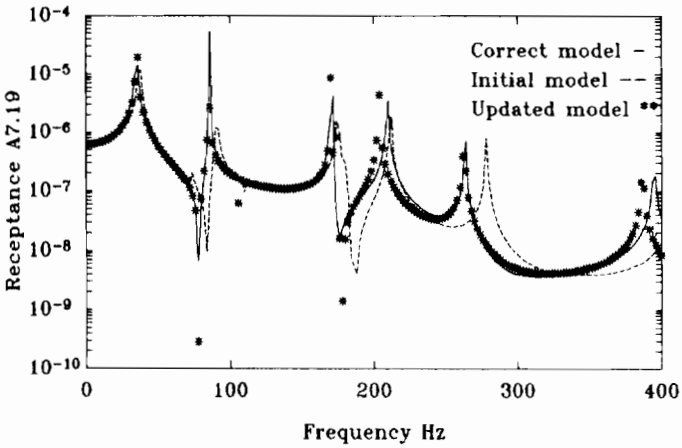


Fig3: Plane frame receptance in the z-DOF at n3
for a force input in the z-DOF at n7.

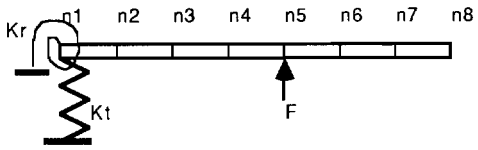


Fig4 Cantilever beam

Element No (No1 at Fixed end)	Updated parameters Case 2.1 EI (NM^2)	Case 2.1 m (Kg/M)	Case 2.2 EI (NM^2)	Case 2.2 m (Kg/M)
1	4500	3.398	4638	3.406
2	4561	3.385	4664	3.453
3	4524	3.379	4368	3.515
4	4384	3.395	4471	3.465
5	4292	3.394	4832	3.371
6	4473	3.333	4850	3.416
7	4561	3.202	4621	3.518
Stiffness Kt(N/M)	8.02E+007		4.02E+007	
Kr(N/M)	2.06E+005		3.17E+005	
Mload (Kg/M)	0.028		0.058	

Table3 Updated parameters-Cantilever beam

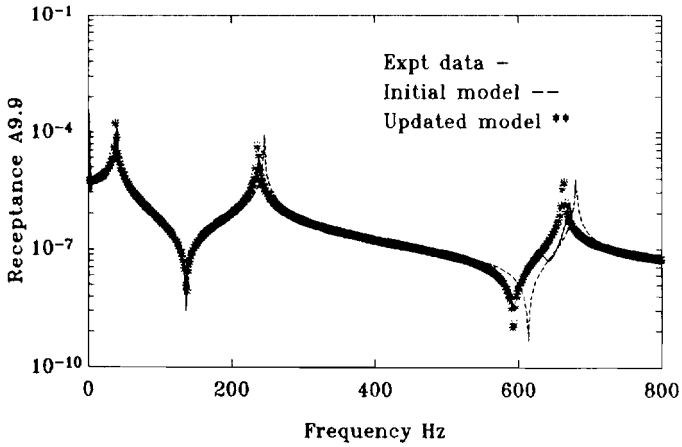


Fig 5. Beam receptance at the excitation coordinate n5.

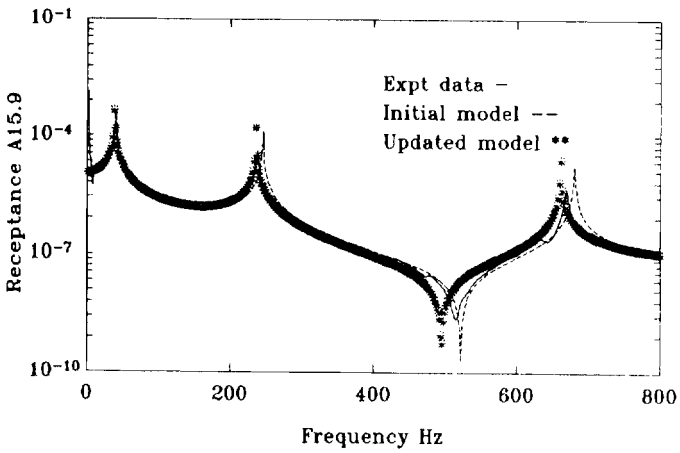


Fig 6. Beam receptance at the tip for a force input at n5.