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Fixed Point Theorem for F-contraction Mappings, in Partial Metric Spaces

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Abstract—The purpose of this paper is to establish a fixed point theorem for F-contraction mappings in partial metric spaces. Also, as a consequence, a fixed point theorem for a pair of F-contraction mappings having a unique common fixed point is obtained. In particular, the main results in this paper generalize and extend a fixed point theorem due to Wardowski 2012 in which F-contraction was introduced as a generalization of Banach Contraction Principle. An illustrative example is provided to validate the results.

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1. INTRODUCTION

The notion of F-contraction mappings was introduced by Wardowski [8] as a generalization of Banach Contraction [5] and proved to be very useful in the existing metric fixed point theory. This gave rise to numerous fixed point results for F-contraction mappings as extension and its generalizations. Wardowski [8] introduced an F-contraction mapping and defined it as follows:

Definition 1. [8] *Let (M,d) be a metric space, a mapping $T : M \longrightarrow M$ is said to be an F-contraction on M, if there exists $\tau > 0$ such that for all $x, y \in M$*

$$d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq F(d(x,y)), \tag{1}$$

and $F : \mathbb{R}_+ \longrightarrow \mathbb{R}$ is a mapping satisfying the following conditions:

$F1$: F is strictly increasing, that is for all $x, y \in \mathbb{R}_+$ such that $x \leq y \Rightarrow F(x) \leq F(y)$;

$F2$: For each sequence $\{\alpha_n\}_{n\geq 1}$ of positive numbers $\lim\limits_{n\to\infty} \alpha_n = 0$, if and only if $\lim\limits_{n\to\infty} F(\alpha_n) = -\infty$;

$F3$: There exists $k \in (0,1)$ such that $\lim\limits_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

We denote the set of all functions satisfying the conditions $F1 - F3$ by Δ_F.

From $F1$ and the contractive condition 1, we observe that every F-contraction is necessarily continuous. The following Example 1 is one amongst the examples of F-contractions [7] :

Example 1. Let $F : \mathbb{R}_+ \longrightarrow \mathbb{R}$ be defined by $F(\alpha) = \ln(\alpha)$. It is clear that F satisfies $F1 - F3$ for any $k \in (0,1)$. Each mapping $T : M \longrightarrow M$ satisfying $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \leq$

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$F(d(x,y))$ is an F-contraction such that $d(Tx,Ty) \leq e^{-\tau}d(x,y)$ for all $x,y \in M, Tx \neq Ty$. Obviously, for all $x,y \in M$ such that $Tx = Ty$, the inequality $d(Tx,Ty) \leq e^{-\tau}d(x,y)$ holds and T is a Banach contraction. One can find more examples in [8].

A partial metric space was introduced by Matthews [6], which is a generalization of metric space and admits non-zero self distance:

Definition 2 [6]. *Let X be a non-empty set. A partial metric space is a pair (X,p), where p is a function $p : X \times X \to \mathbb{R}^+$, called the partial metric, such that for all $x,y,z \in X$ the following axioms hold: (P1): $x = y \Leftrightarrow p(x,y) = p(x,x) = p(y,y)$; (P2): $p(x,x) \leq p(x,y)$; (P3): $p(x,y) = p(y,x)$; and (P4): $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.*

Clearly, by (P1)-(P3), if $p(x,y) = 0$, then $x = y$. But the converse is in general not true.

One among the classical examples of partial metric spaces is a pair $([0,\infty),p)$ where $p(x,y) = \max\{x,y\}$ for all $x,y \in [0,\infty)$. One can find more examples in [3, 6].

Each partial metric p on X generates a T_0 topology τ_p on X whose basis is the collection of all open p-balls $\{B_p(x,\epsilon) : x \in X, \epsilon > 0\}$, where $B_p(x,\epsilon) = \{y \in X : p(x,y) < p(x,x) + \epsilon\}$ for all $x \in X$, and ϵ is a positive real number.

2. PRELIMINARIES

The following are some basic concepts and preliminaries useful for the establishment of our results: Bukatin et al. [3] provided the following definition regarding the convergence and Cauchy sequences in partial metric spaces.

Definition 3 [6]. *Let (X,p) be a partial metric space. Then,*

(i) a sequence $\{x_n\}$ in (X,p) is said to be convergent to $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$;

(ii) a sequence $\{x_n\}$ in (X,p) is a Cauchy sequence if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists and is finite;

(iii) a partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to the topology τ_p to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

Bukatin et al. [3] proved the following lemma which is useful in our discussion.

Lemma 1 [3]. *Let (X,p) be a partial metric space. Then the mapping $p^s : X \times X \to [0,\infty)$ given by $p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ for all $x,y \in X$ defines a metric on X.*

Bukatin et al. [3] also proved the following lemma regarding Cauchy sequence and completeness of a partial metric space.

Lemma 2 [3]. *Let (X,p) be a partial metric space. Then:*

(i) a sequence $\{x_n\}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s); (ii) a partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete.

Some researchers have attempted to generalize, improve and extend the notion of F-contraction mappings to partial metric spaces. For instance Nazam et al. [7] introduced an improved F-contraction of rational type in partial metric spaces and used it to prove a common fixed point theorem for a pair of self mappings. The following definition was given by Nazam et al. 2017 regarding an improved F-contraction of rational type.

Definition 4 (Nazam et al. 2017). *Let (M,p) be a partial metric space. The mapping $T : M \longrightarrow M$ is called an improved F-contraction of rational type, if for all $m_1,m_2 \in M$ we have $\tau + F(p(T(m_1),T(m_2))) \leq F(\mathbb{N}(m_1,m_2))$ for some $F \in \Delta_F$, $\tau > 0$ and*

$$\mathbb{N}(m_1,m_2) = \max\left\{p(m_1,m_2), \frac{p(m_1,T(m_1))p(m_2,T(m_2))}{1+p(m_1,m_2)}, \frac{p(m_1,T(m_1))p(m_2,T(m_2))}{1+p(T(m_1),T(m_2))}\right\}.$$

The following Theorem was given by Nazam et al. [7] regarding F-contraction of rational type.

Theorem 1 [7]. *The contractions $H,T : X \to X$ on a complete partial metric space X such that either H or T is continuous and (H,T) is a pair of improved F-contraction of rational type have a common fixed point z of (H,T) in X such that $P(z,z) = 0$.*

Wardowski [8] proved the following results using F-contraction mapping.

Theorem 2 [8]**.** Example 1. *Let (M,d) be a complete metric space and let $T : M \longrightarrow M$ be an F-contraction on M. Then T has a unique fixed point $x_0 \in M$ and for every $x \in M$ the sequence $\{T^n x\}_{n\in N}$ converges to x_0.*

In this article we generalize Theorem 2 to partial metric spaces to obtain some fixed point theorems for F-contraction mappings in partial metric spaces.

3. MAIN RESULTS

Definition 5. *Let (X,p) be a partial metric space. The mapping $T : X \longrightarrow X$ is said to be an F-contraction on X if there exists $\tau > 0$ and $F \in \Delta_F$ such that for all $x, y \in X$,*

$$p(Tx,Ty) > 0 \Rightarrow \tau + F(p(Tx,Ty)) \leq F(p(x,y)). \tag{2}$$

The following theorem is the extension of Theorem 2 to partial metric spaces.

Theorem 3. *Let (X,p) be a complete partial metric space and let $T : X \longrightarrow X$ be an F-contraction map on X, then T has a unique fixed point $v \in X$ such that $p(v,v) = 0$.*

Proof. Let $x_0 \in X$ be any arbitrary point and fixed. We define a sequence $\{x_n\}_{n\in\mathbb{N}} \in X$ such that $x_{n+1} = Tx_n$, for all $n = 0, 1, 2, ...$ Also we denote $a_n = p(x_{n+1}, x_n)$, for all $n = 0, 1, 2, ...$ If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0+1} = x_{n_0}$, then $Tx_{n_0} = x_{n_0}$. This will end the proof. Now suppose $a_n > 0$ for all $n \in \mathbb{N}$ with $x_{n+1} \neq x_n$. Then by the contractive condition (2) of Definition 5 we have

$$F(a_n) \leq F(a_{n-1}) - \tau \leq F(a_{n-2}) - 2\tau \leq F(a_{n-3}) - 3\tau \leq ... \leq F(a_0) - n\tau. \tag{3}$$

From (3) we obtain $\lim\limits_{n\to\infty} F(a_n) = -\infty$. Since $F \in \Delta_F$, then by $(F2)$ of Definition 1 we get

$$\lim_{n\to\infty} a_n = 0. \tag{4}$$

By $(F3)$ of Definition 1, there exists $k \in (0,1)$ such that

$$\lim_{n\to\infty} {a_n}^k F(a_n) = 0. \tag{5}$$

Following (3) for all $n \in \mathbb{N}$ we have

$${a_n}^k (F(a_n) - F(a_0)) \leq -{a_n}^k n\tau \leq 0. \tag{6}$$

Taking into account (4) and (5) and letting $n \to \infty$ in (6) we obtain

$$\lim_{n\to\infty} n{a_n}^k = 0. \tag{7}$$

Since (7) holds, then there exists $n_1 \in \mathbb{N}$ such that $n{a_n}^k \leq 1$, for all $n \geq n_1$. This implies that

$$a_n \leq n^{-1/k} \text{ for all } n \geq n_1. \tag{8}$$

Next we show that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence.

Consider $n, m \in \mathbb{N}$ such that $m > n \geq n_1$, then by (8) and axiom $(P3)$ of Definition 2 we have

$$\begin{aligned} p(x_n,x_m) &\leq p(x_n,x_{n+1}) + p(x_{n+1},x_{n+2}) + ... + p(x_{m-1},x_m) - \sum_{j=n+1}^{m-1} p(x_j,x_j) \\ &\leq p(x_n,x_{n+1}) + p(x_{n+1},x_{n+2}) + ... + p(x_{m-1},x_m) = a_n + a_{n+1} + ... + a_{m-1} \\ &= \sum_{i=n}^{m-1} a_i \leq \sum_{i=n}^{\infty} a_i \leq \sum_{i=n}^{\infty} i^{-1/k}. \end{aligned}$$

The convergence of the series $\sum_{i=n}^{\infty} i^{-1/k}$ implies that $\lim\limits_{n\to\infty} p(x_n,x_m) = 0$. By Lemma 1 we obtain that for any $n, m \in \mathbb{N}$, $p^s(x_n,x_m) \leq 2p(x_n,x_m) \to 0$ as $n \to \infty$. This implies that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence with respect to p^s and hence converges by Lemma 2. Thus there exists $v \in X$ such that

$$p(v,v) = \lim_{n\to\infty} p(x_n,v) = \lim_{n,m\to\infty} p(x_n,x_m).$$

Since $\lim_{n,m\to\infty} p(x_n, x_m) = 0$, then we deduce that

$$p(v, v) = 0 = \lim_{n\to\infty} p(x_n, v). \tag{9}$$

By (9) it follows that $x_{n+1} \to v$ as $n \to \infty$ with respect to τ_p. By the continuity of T it implies that

$$v = \lim_{n\to\infty} x_n = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} Tx_n = Tv.$$

From the contractive condition (2) we can write $\tau + F(p(v, Tv)) \leq F(p(v, v))$. This implies that $p(v, Tv) = 0$ and by axioms $(P1)$ and $(P2)$ of Definition 2 we conclude that $v = Tv$. Hence v is a fixed point of T. We now prove that a fixed point is unique.

Suppose, there exists $u \in X$ such that $u \neq v$ and $u = Tu$. From the contractive condition (2) we can have

$$p(Tu, Tv) = p(u, v) > 0 \Rightarrow F(p(u, v)) < \tau + F(p(u, v)) = \tau + F(p(Tu, Tv)) \leq F(p(u, v)).$$

This implies that $F(p(u, v)) < F(p(u, v))$ which is a contradiction. Hence $u = v$. Thus the fixed point of T is unique. □

Corollary. *Let (X, p) be a complete partial metric space and a contraction $T : X \to X$ satisfies all conditions in Theorem 3. If we take F as defined in Example 1, then T is a Banach Contraction as generalized by Matthews* [6].

We now present a fixed point theorem for a pair of maps satisfying F-contraction condition as an extension of Theorem 3.

Theorem 4. *Let (X, p) be a complete partial metric space and $H, T : X \to X$ be a pair of F-contraction mappings, such that for all $x, y \in X$ we have*

$$p(Hx, Ty) > 0 \Rightarrow \tau + F(p(Hx, Ty)) \leq F(\mathbb{M}(x, y)),$$

where

$$\mathbb{M}(x, y) = \max\left\{p(x, y), p(x, Hx), p(y, Ty), \frac{p(x, Ty) + p(y, Hx)}{2}\right\}. \tag{10}$$

Then there exists a unique fixed point which is common for both H and T.

Proof. We first show the existence of a fixed point for both H and T. Let $x_0 \in X$ be any arbitrary point and fixed. We define a sequence $\{x_n\} \in X$, for all $n \in \mathbb{N}$ such that $x_{n+1} = Hx_n$ and $x_{n+2} = Tx_{n+1}$ for $n = 0, 1, 2, \ldots$.

Now suppose that $p(x_{n+1}, x_{n+2}) > 0$, for all $n \in \mathbb{N} \cup \{0\}$ with $x_{n+1} \neq x_{n+2}$. Then by the contractive condition (10) we have

$$\begin{aligned}
&\tau + F(p(x_{n+1}, x_{n+2})) = \tau + F(p(Hx_n, Tx_{n+1})) \leq F(\mathbb{M}(x_n, x_{n+1})),\\
\mathbb{M}(x_n, x_{n+1}) &= \max\left\{p(x_n, x_{n+1}), p(x_n, Hx_n), p(x_{n+1}, Tx_{n+1}), \tfrac{p(x_n, Tx_{n+1}) + p(x_{n+1}, Hx_n)}{2}\right\}\\
&= \max\left\{p(x_n, x_{n+1}), p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2}), \tfrac{p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})}{2}\right\}\\
&= \max\left\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2}), \tfrac{p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2})}{2}\right\}\\
&= \max\left\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\right\}.
\end{aligned}$$

Suppose $\max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\} = p(x_{n+1}, x_{n+2})$. Then

$$\tau + F(p(x_{n+1}, x_{n+2})) \leq F(p(x_{n+1}, x_{n+2})) \Rightarrow F(p(x_{n+1}, x_{n+2})) \leq F(p(x_{n+1}, x_{n+2})) - \tau$$

which is a contradiction. Thus $\max\{p(x_n, x_{n+1}), p(x_{n+1}, x_{n+2})\} = p(x_n, x_{n+1})$. Hence we can write

$$F(p(x_{n+1}, x_{n+2})) \leq F(p(x_n, x_{n+1})) - \tau \text{ for all } n \in \mathbb{N} \cup \{0\}. \tag{11}$$

Hence from (11) we have $F(p(x_n, x_{n+1})) \leq F(p(x_{n-1}, x_{n-2})) - 2\tau$. Repeating these steps n-times we get $F(p(x_n, x_{n+1})) \leq F(p(x_0, x_1)) - n\tau$ and we obtain

$$\lim_{n\to\infty} F(p(x_n, x_{n+1})) = -\infty. \tag{12}$$

By $(F1)$ of Definition 1 we obtain

$$\lim_{n\to\infty} (p(x_n, x_{n+1})) = 0. \tag{13}$$

By $(F3)$, there exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} (p(x_n, x_{n+1}))^k F(p(x_n, x_{n+1})) = 0. \tag{14}$$

For all $n \in \mathbb{N}$ we have

$$(p(x_n, x_{n+1}))^k (F(p(x_n, x_{n+1})) - F(p(x_0, x_1))) \leq -(p(x_n, x_{n+1}))^k n\tau \leq 0.$$

Considering (12), (13) and letting $n \to \infty$ in (14), we get $\lim\limits_{n\to\infty} n(p(x_n, x_{n+1}))^k = 0$, and there exists $n_1 \in \mathbb{N}$, such that

$$n(p(x_n, x_{n+1}))^k \leq 1 \text{ for all } n \geq n_1. \tag{15}$$

Next we will show that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence. Consider $n, m \in \mathbb{N}$ such that $m > n \geq n_1$, then by (15) and axiom $(P3)$ of Definition 2 we have

$$\begin{aligned} p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + ... + p(x_{m-1}, x_m) - \sum_{j=n+1}^{m-1} p(x_j, x_j) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + ... + p(x_{m-1}, x_m) \\ &= a_n + a_{n+1} + ... + a_{m-1} = \sum_{i=n}^{m-1} a_i \leq \sum_{i=n}^{\infty} a_i \leq \sum_{i=n}^{\infty} i^{-1/k}. \end{aligned}$$

The convergence of the series $\sum_{i=n}^{\infty} i^{-1/k}$ implies that $\lim\limits_{n\to\infty} p(x_n, x_m) = 0$. By Lemma 1 we get that for any $n, m \in \mathbb{N}, p^s(x_n, x_m) \leq 2p(x_n, x_m) \to 0$ as $n \to \infty$. This implies that $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence with respect to p^s. Since (X, p) is complete then so is (X, p^s), then there exists $u \in X$ such that $\lim\limits_{n\to\infty} p^s(x_n, u) = 0$. Moreover by Lemma 2 $p(u, u) = \lim\limits_{n\to\infty} p(x_n, u) = \lim\limits_{n,m\to\infty} p(x_n, x_m)$. Then we deduce that $p(u, u) = 0 = \lim\limits_{n\to\infty} p(x_n, u)$. It follows that $x_{n+1} \to u$ and $x_{n+2} \to u$ as $n \to \infty$ with respect to $\tau(p)$. Hence by the continuity of T it implies that $u = \lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} x_{n+1} = \lim\limits_{n\to\infty} x_{n+2} = \lim\limits_{n\to\infty} Tx_{n+1} = T \lim\limits_{n\to\infty} x_{n+1} = Tu$. Hence from (10) we have

$$\tau + F(p(u, Hu)) = \tau + F(p(Hu, Tu)) \leq F(\mathbb{M}(u, u)) = F(p(u, u)).$$

This yields that $p(u, Hu) = 0$ and by $(P1)$, $(P2)$ of Definition 2 we obtain that $u = Hu$. Thus $Hu = Tu = u$. Hence (H, T) has a common fixed point $u \in X$.

Next we will show that u is unique common fixed point of H and T. Suppose that by contradiction there exists $z \in X$ such that $u \neq z$ and $z = Tz$. From the contractive condition (10) we have $\tau + F(p(Hu, Tz)) \leq F(\mathbb{M}(u, z))$, where

$$\begin{aligned} \mathbb{M}(u, z) &= \max\left\{p(u, z), p(u, Hu), p(z, Tz), \frac{p(u, Tz) + p(z, Hu)}{2}\right\} \\ &= \max\left\{p(u, z), p(u, u), p(z, z), \frac{p(u, z) + p(z, u)}{2}\right\} = \max\left\{p(u, z), \frac{p(u, z) + p(z, u)}{2}\right\} = p(u, z). \end{aligned}$$

We have $\tau + F(p(Hu, Tz)) \leq F(p(u, z))$ and we obtain that $p(u, z) < p(u, z)$ which is a contradiction. Hence $u = z$ and u is a unique common fixed point of (H, T). □

Now we provide an illustrative Example 2 for Theorem 3.

Example 2. Let $X = [0, 1]$ and define $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a complete partial metric space. Define the mapping $T : X \to X$ such that for all $x \in X$

$$T(x) = \begin{cases} x/3, & \text{if} \quad x \in [0, 1), \\ 0, & \text{if} \quad x = 1. \end{cases}$$

Clearly T is a self mapping. Define the function $F : \mathbb{R}^+ \to \mathbb{R}$ by $F(r) = \ln(r)$ for all $r \in \mathbb{R}^+$. Let $x, y \in X$ such that $p(Tx, Ty) > 0$, this implies that $\tau + F(p(Tx, Ty)) = \tau + \ln \max\{x/3, y/3\}$. Now suppose that $y \geq x$ without loss of generality and taking $\tau \leq \ln 3$ we obtain that

$$\tau + \ln \max\{x/3, y/3\} \leq \ln 3 + \ln(y/3) = \ln p(x, y) = F(p(x, y)).$$

Similarly, if $x \geq y$ we obtain that $\tau + F(p(Tx, Ty)) \leq F(p(x, y))$. Thus, the contractive condition (2) is satisfied for all $x, y \in X$. Hence all hypotheses of the Theorem 3 are satisfied and note that T has a unique fixed point $x = 0$. We also notice that T in Example 2 is not an F-contraction in (X, d) and consequently Theorem 3 cannot be applied.

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REFERENCES

1. J. Ahmad, A. Al-Rawashdeh, and A. Azam, "New fixed point theorems for generalized F-contractions in complete metric spaces," Fixed Point Theory Appl. **1**, 80 (2015).
2. H. Aydi, M. Abbas, and C. Vetro, "Partial Hausdorff metric and Nadler's fixed point theorem on partial metric spaces," Topol. Appl. **159**, 3234–3242 (2012).
3. M. Bukatin, R. Kopperman, and S. Matthews, "Partial metric spaces," Am. Math. Mon. **116**, 708–718 (2009).
4. S. Chang, J. Kim, L. Wang, and J. Tang, "Existence of fixed points for generalized F-contractions and generalized F-Suzuki contractions in metric spaces," Global J. Pure Appl. Math. **12**, 4867–4882 (2016).
5. M. Khamsi and W. Kirk, *An Introduction to Metric Spaces and Fixed Point Theory,* Vol. 53 of *Pure and Applied Mathematics* (Wiley, New York, 2011).
6. S. Matthews, "Partial metric topology in papers on general topology and applications," in *Proceedings of the 8th Summer Conference at Queens College,* Ed. by S. Andima et al., Ann. N.Y. Acad. Sci. **728**, 183–197 (1992).
7. M. Nazam, M. Arshad, and M. Abbas, "Existence of common fixed points of improved F-contraction on partial metric spaces," Appl. Gen. Topol. **18**, 277–287 (2017).
8. D. Wardowski, "Fixed points of a new type of contractive mappings in complete metric spaces," Fixed Point Theory Appl. **94** (2012).