IDENTIFICATION OF UNBALANCED FORCES AND FOUNDATION PARAMETERS OF ROTATING MACHINES FROM VIBRATION MEASUREMENTS

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Abstract

A technique is described to use measured vibration data of a rotating machine and its foundation to identify unbalanced forces, stiffness and damping parameters of the mountings, and the parameters of the foundation. It is based on an idealisation treating the rotor, the machine structure and the foundation as rigid masses supported by springs and dampers. Operational vibration data of the machine and its foundation before and after the rotating unbalanced forces are perturbed by adding unbalanced mass to the rotor are used in the identification procedure. Once the parameters are identified, dynamic forces transmitted to the foundation can be estimated. The technique is demonstrated using simulated example for a machine with a two bearings rotor.

*[**Keywords:** Identification, unbalanced forces, rotating machines, vibration, foundation]*

1.0 INTRODUCTION

Rotating machines are liable to shaking as a result of excitations due to unbalanced dynamic forces and moments generated within the machines. These unbalanced excitations ultimately find their way to the foundations or supporting structures. With unfavourable combination of operating speed, stiffness and damping properties of the mounting system, amplification of the transmitted forces may occur. The transmissibility of large dynamic force to the foundation is always an issue of concern due to possible structural failure of the foundation or disturbing vibrations being transmitted to nearby structures or occupants. It is known, however, that balancing may not be perfect and in some cases it is not practical to balance the forces fully. Furthermore, isolation systems are usually designed on the assumption of a fixed foundation, whereas in reality a foundation may be flexible and its dynamic behaviour unknown. Apart from the need to design machinery to operate within acceptable limits, accurate dynamic modelling of machine systems is important. Dynamic models may be used to facilitate diagnosis of operational difficulties. The analysis of vibration behaviour of turbo-machinery, for example, is a topic of great importance in most process industries and particularly in power generation. A number of authors have addressed this problem (Lees (1988), Zanetta (1992), Odiara and Ewins (1992), Lees and Friswell (1996), Sinha *et al* (2002)) and the conclusion is that the supporting structure of a large turbine presents a significant problem, having a significant effect on the dynamic behaviour of the machine. The previous researchers have attempted to establish foundation's dynamic properties by extracting information from the response of the foundation due to a known unbalance on the rotor. The key to the problem was the derivation of forces exerted on the foundations at the bearings. Lees and Friswell (1996), and Sinha *et al* (2002) use vibration measurements of the bearing pedestals to derive the forces transmitted to the foundation. Their methods assume the bearing stiffness is known and the foundation is modelled as a mass less spring. This paper presents an alternative technique which does not assume prior knowledge of the rotor unbalance or bearing stiffness and the foundation is modelled as a mass-spring-damper system. The vibration response of the machine and the foundation before and after the unbalanced forces are perturbed by adding unbalanced mass to the rotor are used in the identification. Unbalanced forces, stiffness and damping properties of the

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machine mounting system as well as mass, stiffness and damping properties of the foundation are identified. The technique is demonstrated using numerical example on a machine with a two bearings rotor.

2.0 THEORY

2.1 MATHEMATICAL MODEL

Consider a rotating machine with a 2 bearings rotor as shown in Fig. 1. The rotor, treated as rigid, is supported by two bearings of stiffness k_{b1} and k_{b2} and damping coefficients c_{b1} and c_{b2} respectively. The rotor is of mass m_r and mass moment of inertia I_r about the centre of gravity of the rotor and in the vertical plane shown. The machine structure is also treated as rigid and is mounted to the foundation through mounting stiffness k_{s1} and k_{s2} and damping coefficients c_{s1} and c_{s2}. The machine structure is of mass m and mass moment of inertia I. about its centre of gravity. The foundation is modelled as a mass-spring-damper system with respective parameters m_f, k_f and c_f being the same under both mountings. The system possesses 6 degrees-of-freedom (DOF) labelled as q_1 to q_6.

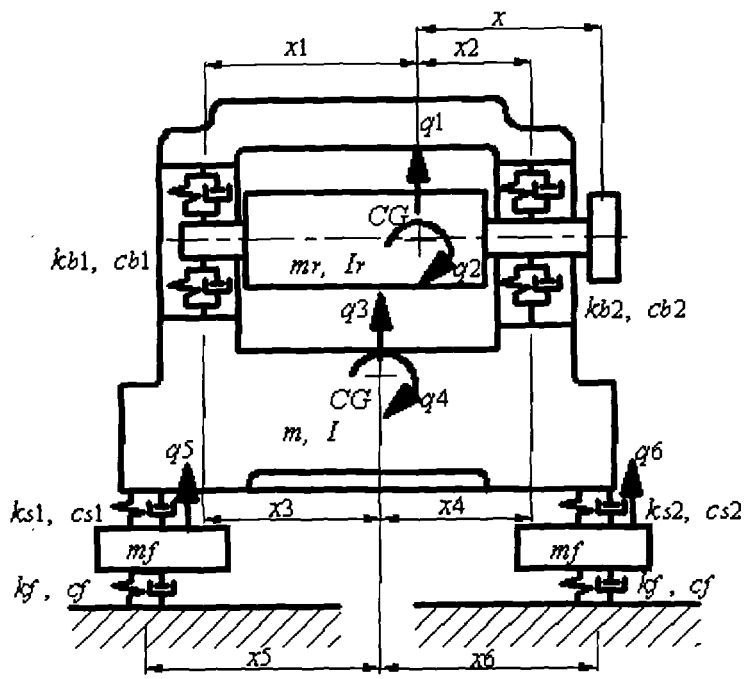


Fig. 1: Model of a Rotating Machine with a Two Bearings Rotor

Let the rotor unbalance be represented by unbalance force $F_u(t)$ and moment $M_u(t)$ at its centre of gravity. These unknown unbalanced loads result in bearing forces $F_{b1}(t)$ and $F_{b2}(t)$ being transmitted from the rotor to the machine structure through the two rotor bearings. Fig. 2(a) shows the model of the rotor and the machine structure, with the rotor isolated from the machine, depicting the free body diagram. Fig. 2(b) is an equivalent model of the machine structure where the bearing loads are represented by an equivalent unbalanced force $F_B(t)$ and moment $M_B(t)$. Distances x_1, x_2 are horizontal distances to the bearings from the rotor centre of gravity whereas x_3 and x_4 are horizontal distances to the bearings from the centre of gravity of the machine structure. Distances x_5 and x_6 are horizontal distances of the machine mountings to the centre of gravity of the machine.

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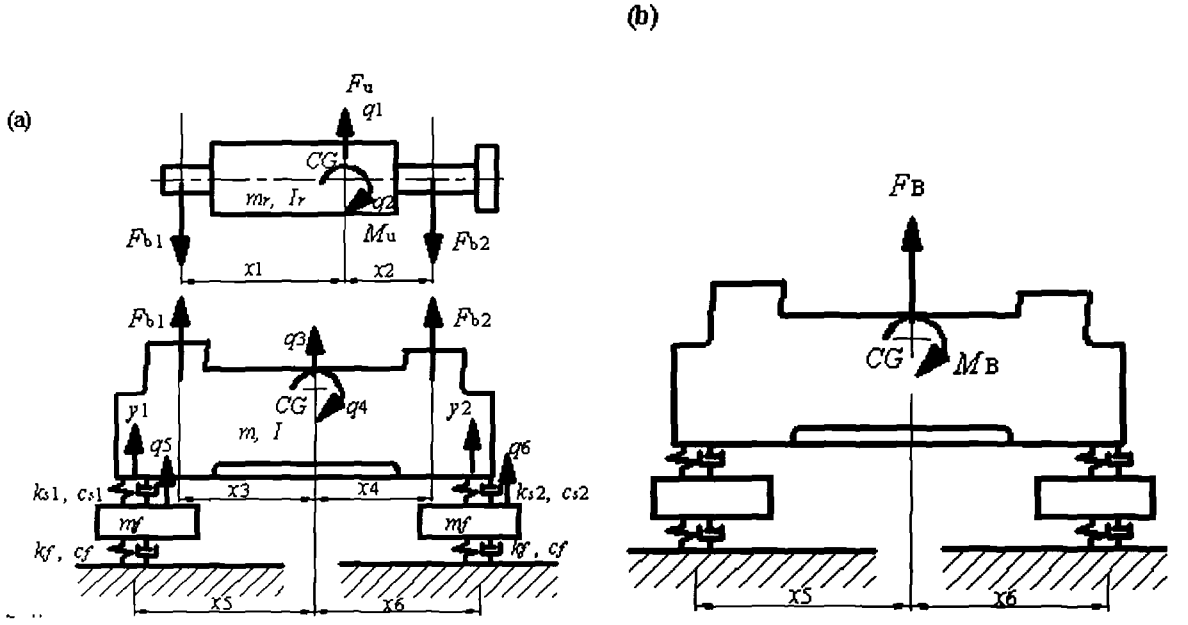


Fig. 2: (a) Free Body Diagrams (b) Equivalent loading on Machine Structure

Applying Newton's equation of motion to the rotor in the translational degree-of-freedom results in the following:

$$F_u(t) - F_{b1}(t) - F_{b2}(t) = m_r \ddot{q}_1(t)$$

$$F_{b1}(t) + F_{b2}(t) = F_u(t) - m_r \ddot{q}_1(t) \qquad (1)$$

Likewise, Newton's equation of motion applied to the machine structure in the translational degree-of-freedom results in:

$$F_{b1}(t) + F_{b2}(t) - k_{s1}[y_1(t) - q_5(t)] - c_{s1}[\dot{y}_1(t) - \dot{q}_5(t)] - k_{s2}[y_2(t) - q_6(t)] - c_{s2}[\dot{y}_2(t) - \dot{q}_6(t)] = m\ddot{q}_3(t) \qquad (2)$$

Replacing $F_{b1}(t) + F_{b2}(t)$ in eq. (2) by eq. (1) and re-arranging results in:

$$k_{s1}[y_1(t) - q_5(t)] + c_{s1}[\dot{y}_1(t) - \dot{q}_5(t)] + k_{s2}[y_2(t) - q_6(t)] + c_{s2}[\dot{y}_2(t) - \dot{q}_6(t)] - F_u(t) = -m_r \ddot{q}_1(t) - m\ddot{q}_3(t) \qquad (3)$$

Let the unbalanced force and moment be harmonic. The resulting vibrations will also be harmonic. Therefore

$$F_u(t) = |F_u|\cos(\omega t + \varphi_f) = \text{Real}(F_u e^{j(\omega t + \varphi_f)})$$

$$M_u(t) = |M_u|\cos(\omega t + \varphi_m) = \text{Real}(M_u e^{j(\omega t + \varphi_m)})$$

$$q_i(t) = |q_i|\cos(\omega t + \varphi_i) = \text{Real}(q_i e^{j(\omega t + \varphi_i)}) \qquad (i = 1 \ldots 6)$$

$$y_i(t) = |y_i|\cos(\omega t + \varphi_i) = \text{Real}(y_i e^{j(\omega t + \varphi_i)}) \qquad (i = 1 \ldots 2)$$

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In general, F_u, M_u, q_i and y_i are complex quantities with the phase angles referred from an arbitrary datum. In view of the above, eq. (3) can be transformed into the frequency domain as:

$$(k_{s1} + j\omega c_{s1})(y_1 - q_5) + (k_{s2} + j\omega c_{s2})(y_2 - q_6) - F_u = \omega^2 m_r q_1 + \omega^2 m q_3 \quad (4)$$

Newton's equation of motion applied to the foundation under the two supports results in the following two equations:

$$-k_f q_5(t) - c_f \dot{y}_5(t) + k_{s1}[y_1(t) - q_5(t)] + c_{s1}[\dot{y}_1(t) - \dot{q}_5(t)] = m_f \ddot{q}_5(t)$$

$$-k_f q_6(t) - c_f \dot{y}_2(t) + k_{s2}[y_2(t) - q_6(t)] + c_{s2}[\dot{y}_2(t) - \dot{q}_6(t)] = m_f \ddot{q}_6(t)$$

Transforming them into the frequency domain in terms of complex displacements, and re-arranging, result in:

$$-\omega^2 m_f q_5 + (k_f + j\omega c_f) q_5 = (k_{s1} + j\omega c_{s1})(y_1 - q_5) \quad (5)$$

and

$$-\omega^2 m_f q_6 + (k_f + j\omega c_f) q_6 = (k_{s2} + j\omega c_{s2})(y_2 - q_6) \quad (6)$$

Now consider moment equilibrium of the rotor in the plane shown and about the centre of gravity of the machine structure.

$$M_u(t) + (x_3 - x_1)F_u(t) + x_4 F_{b2}(t) - x_3 F_{b1}(t) = I_r \ddot{q}_2(t) + (x_3 - x_1) m_r \ddot{q}_1(t)$$

Re-arranging:

$$x_3 F_{b1}(t) - x_4 F_{b2}(t) = M_u(t) + (x_3 - x_1)F_u(t) - I_r \ddot{q}_2(t) - (x_3 - x_1) m_r \ddot{q}_1(t) \quad (7)$$

Similarly, consider moment equilibrium of the machine structure in the plane shown and about its centre of gravity:

$$x_3 F_{b1}(t) - x_4 F_{b2}(t) + k_{s2}[y_2(t) - q_6(t)]x_6 + c_{s2}[\dot{y}_2(t) - \dot{q}_6(t)]x_6 - k_{s1}[y_1(t) - q_5(t)]x_5 - c_{s1}[\dot{y}_1(t) - \dot{q}_5(t)]x_5 = I\ddot{q}_4(t) \quad (8)$$

Replacing $x_3 F_{b1}(t) - x_4 F_{b2}(t)$ in eq. (8) by eq. (7) results in:

$$M_u(t) + (x_3 - x_1)F_u(t) + k_{s2}[y_2(t) - q_6(t)]x_6 + c_{s2}[\dot{y}_2(t) - \dot{q}_6(t)]x_6 - k_{s1}[y_1(t) - q_5(t)]x_5 - c_{s1}[\dot{y}_1(t) - \dot{q}_5(t)]x_5 = I\ddot{q}_4(t) + I_r \ddot{q}_2(t) + (x_3 - x_1) m_r \ddot{q}_1(t) \quad (9)$$

Since the unbalanced force and moment as well as the resulting vibrations are harmonic, eq. (9) can be expressed in terms of complex quantities and on re-arranging results in:

$$-(k_{s1} + j\omega c_{s1})(y_1 - q_5)x_5 + (k_{s2} + j\omega c_{s2})(y_2 - q_6)x_6 + (x_3 - x_1)F_u + M_u = \omega^2 m_r q_1 (x_1 - x_3) - \omega^2 I_r q_2 - \omega^2 I q_4 \quad (10)$$

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Now, eqs. (4) and (10) are the key translational and rotational equations of motion needed for the next stage of identification of unbalanced force, moment and mounting parameters. Eqs. (5) and (6) shall be used to identify foundation parameters once the mounting parameters are identified.

2.2 IDENTIFICATION OF FORCES AND MOUNTING PARAMETERS

Suppose we now perturb the unbalanced force by adding known unbalanced mass Δm to the rotor and with this added mass, we measure new vibrations which we assign a subscript p to indicate measurement on the perturbed system. Let e denote the radial position vector to this added mass such that the increment in unbalanced force due to the added mass is $\omega^2 \Delta m \times e$. Thus, for this perturbed case, eq. (4) becomes:

$$(k_{s1} + j\omega c_{s1})(y_{1p} - q_{5p}) + (k_{s2} + j\omega c_{s2})(y_{2p} - q_{6p}) - F_u = \omega^2 m_r q_{1p} + \omega^2 m q_{3p} + \omega^2 \Delta m \times e \quad (11)$$

Let x_e be the moment arm perpendicular distance of the line of action of the perturbing centrifugal force from the centre of gravity of the machine structure. The added mass will result in moment increment of $\omega^2 \Delta m \times e x_e$ Therefore, moment equation, eq. (10), becomes:

$$-(k_{s1} + j\omega c_{s1})(y_{1p} - q_{5p})x_5 + (k_{s2} + j\omega c_{s2})(y_{2p} - q_{6p})x_6 + (x_3 - x_1)F_u + M_u$$

$$= \omega^2 m_r q_{1p}(x_1 - x_3) - \omega^2 I_r q_{2p} - \omega^2 I q_{4p} - (x_3 - x_1)\omega^2 \Delta m \times e - \omega^2 \Delta m \times e x_e \quad (12)$$

Equations (4), (10), (11) and (12) are combined into the following matrix equation:

$$[\mathbf{Q}] \begin{Bmatrix} k_{s1} + j\omega c_{s1} \\ k_{s2} + j\omega c_{s2} \\ F_u \\ M_u \end{Bmatrix} = \{\mathbf{b}\} \quad (13)$$

where

$$[\mathbf{Q}] = \begin{bmatrix} (y_1 - q_5) & (y_2 - q_6) & -1 & 0 \\ -(y_1 - q_5)x_5 & (y_2 - q_6)x_6 & (x_3 - x_1) & 1 \\ (y_{1p} - q_{5p}) & (y_{2p} - q_{6p}) & -1 & 0 \\ -(y_{1p} - q_{5p})x_5 & (y_{2p} - q_{6p})x_6 & (x_3 - x_1) & 1 \end{bmatrix}$$

$$\{\mathbf{b}\} = \begin{Bmatrix} \omega^2 m_r q_1 + \omega^2 m q_3 \\ \omega^2 m_r q_1(x_1 - x_3) - \omega^2 I_r q_2 - \omega^2 I q_4 \\ \omega^2 m_r q_{1p} + \omega^2 m q_{3p} + \omega^2 \Delta m \times e \\ \omega^2 m_r q_{1p}(x_1 - x_3) - \omega^2 I_r q_{2p} - \omega^2 I q_{4p} - (x_3 - x_1)\omega^2 \Delta m \times e - \omega^2 \Delta m \times e x_e \end{Bmatrix}$$

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We can perform more than one case of mass additions on the rotor and the corresponding vibrations used to build up [**Q**] and {b} matrices. For a rectangular [**Q**] matrix, the unknown parameters are obtained by pseudo-inverse solution of eq. (13). Thus,

$$\begin{Bmatrix} k_{s1} + j\omega c_{s1} \\ k_{s2} + j\omega c_{s2} \\ F_u \\ M_u \end{Bmatrix} = \left[\mathbf{Q}^{\mathrm{T}}\mathbf{Q}\right]^{-1}\mathbf{Q}^{\mathrm{T}}\{\mathbf{b}\} \tag{14}$$

2.3 IDENTIFICATION OF FOUNDATION PARAMETERS

Adding unbalanced mass to the rotor employed in section 2.1 cannot help us to solve for the foundation parameters in eqs. (5) and (6). Instead, after establishing mounting parameters k_{s1}, c_{s1}, k_{s2} and c_{s2} from eq. (14), the rotor is operated at a different speed, resulting in new vibrations. Let subscript x be added to denote frequency and displacements corresponding to the new speed. At this new speed, eq. (5) becomes:

$$-\omega_x{}^2 m_f q_{5x} + (k_f + j\omega_x c_f)q_{5x} = (k_{s1} + j\omega_x c_{s1})(y_{1x} - q_{5x}) \tag{15}$$

Multiplying eq. (5) by $\omega_x{}^2 q_{5x}$ and eq. (15) by $\omega^2 q_5$ respectively, result in:

$$-\omega_x{}^2\omega^2 m_f q_5 q_{5x} + (k_f + j\omega c_f)q_5\omega_x{}^2 q_{5x} = (k_{s1} + j\omega c_{s1})(y_1 - q_5)\omega_x{}^2 q_{5x} \tag{16}$$

$$-\omega^2\omega_x{}^2 m_f q_{5x} q_5 + (k_f + j\omega_x c_f)q_{5x}\omega^2 q_5 = (k_{s1} + j\omega_x c_{s1})(y_{1x} - q_{5x})\omega^2 q_5 \tag{17}$$

Subtract eq. (16) from eq. (17) results in:

$$k_f(\omega^2 - \omega_x{}^2)q_5 q_{5x} + jc_f(\omega^2\omega_x - \omega_x{}^2\omega)q_5 q_{5x} = A_1 + jB_1 \tag{18}$$

Where:

$$A_1 + jB_1 = (k_{s1} + j\omega_x c_{s1})(y_{1x} - q_{5x})\omega^2 q_5 - (k_{s1} + j\omega c_{s1})(y_1 - q_5)\omega_x{}^2 q_{5x}$$

Eq. (18) is transformed into two separate equations by equating the real parts as well as imaginary parts. Thus:

$$(\omega^2 - \omega_x{}^2)k_f \mathrm{Real}(q_5 q_{5x}) - (\omega^2\omega_x - \omega_x{}^2\omega)c_f \mathrm{Imag}(q_5 q_{5x}) = A_1 \tag{19}$$

$$(\omega^2 - \omega_x{}^2)k_f \mathrm{Imag}(q_5 q_{5x}) + (\omega^2\omega_x - \omega_x{}^2\omega)c_f \mathrm{Real}(q_5 q_{5x}) = B_1 \tag{20}$$

Likewise, the other mounting support results in similar equations but based on vibrations q_6 and q_{6x} at rotor frequencies

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Equations (19) to (22) may be used to solve for k_f and c_f. However, before proceeding with the solution for k_f and c_f, lets first consider m_f. Multiplying eqs. (5) and (15) by q_{5x} and q_5 respectively, result in:

$$-\omega^2 m_f q_5 q_{5x} + (k_f + j\omega c_f) q_5 q_{5x} = (k_{s1} + j\omega c_{s1})(y_1 - q_5) q_{5x} \tag{23}$$

and

$$-\omega_x{}^2 m_f q_{5x} q_5 + (k_f + j\omega_x c_f) q_{5x} q_5 = (k_{s1} + j\omega_x c_{s1})(y_{1x} - q_{5x}) q_5 \tag{24}$$

Subtracting eq. (23) from eq. (24) results in:

$$(\omega^2 - \omega_x{}^2) m_f q_5 q_{5x} - j(\omega - \omega_x) c_f q_5 q_{5x} = k_{s1}[(y_{1x} - q_{5x}) q_5 - (y_1 - q_5) q_{5x}] + jc_{s1}[(y_{1x} - q_{5x})\omega_x q_5 - (y_1 - q_5)\omega q_{5x}]$$

Or

$$-j(\omega - \omega_x) c_f q_5 q_{5x} + (\omega^2 - \omega_x{}^2) m_f q_5 q_{5x} = C_1 + jD_1 \tag{25}$$

Where,

$$C_1 + jD_1 = k_{s1}[(y_{1x} - q_{5x}) q_5 - (y_1 - q_5) q_{5x}] + jc_{s1}[(y_{1x} - q_{5x})\omega_x q_5 - (y_1 - q_5)\omega q_{5x}]$$

Equating real parts and also imaginary parts of eq. (25) results in two equations as follows:

$$(\omega - \omega_x) c_f \text{Imag}(q_5 q_{5x}) + (\omega^2 - \omega_x{}^2) m_f \text{Real}(q_5 q_{5x}) = C_1 \tag{26}$$

$$-(\omega - \omega_x) c_f \text{Real}(q_5 q_{5x}) + (\omega^2 - \omega_x{}^2) m_f \text{Imag}(q_5 q_{5x}) = D_1 \tag{27}$$

Similarly, the equation of motion for the second support side results in similar expressions but with vibrations q_6 and y_2 instead of q_5 and y_1, and $C_1 + jD_1$ replaced by a corresponding vector $C_2 + jD_2$. Thus:

$$(\omega - \omega_x) c_f \text{Imag}(q_6 q_{6x}) + (\omega^2 - \omega_x{}^2) m_f \text{Real}(q_6 q_{6x}) = C_2 \tag{28}$$

$$-(\omega - \omega_x) c_f \text{Real}(q_6 q_{6x}) + (\omega^2 - \omega_x{}^2) m_f \text{Imag}(q_6 q_{6x}) = D_2 \tag{29}$$

Where:

$$C_2 + jD_2 = k_{s2}[(y_{2x} - q_{6x}) q_6 - (y_2 - q_6) q_{6x}] + jc_{s2}[(y_{2x} - q_{6x})\omega_x q_6 - (y_2 - q_6)\omega q_{6x}]$$

Equations (19) – (22) are now combined with eqs. (26) - (29) to become a set of 8 simultaneous equations with three unknown parameters, i.e. k_f, c_f and m_f. These equations are reformulated into a matrix equation, and solve for the three unknown parameters by pseudo-inverse technique as shown in eq. (30), where **R** is an 8x3-coefficient matrix. Thus,

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$$[\mathbf{R}]\begin{Bmatrix} k_f \\ c_f \\ m_f \end{Bmatrix} = \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ C_1 \\ D_1 \\ C_2 \\ D_2 \end{Bmatrix}; \qquad \text{Hence,} \quad \begin{Bmatrix} k_f \\ c_f \\ m_f \end{Bmatrix} = [\mathbf{R}^T\mathbf{R}]^{-1}\mathbf{R}^T \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ C_1 \\ D_1 \\ C_2 \\ D_2 \end{Bmatrix} \tag{30}$$

Where,

$$[\mathbf{R}] = \begin{bmatrix} (\omega^2 - \omega_x{}^2)\text{Real}(q_5 q_{5x}) & -(\omega^2\omega_x - \omega_x{}^2\omega)\text{Imag}(q_5 q_{5x}) & 0 \\ (\omega^2 - \omega_x{}^2)\text{Imag}(q_5 q_{5x}) & (\omega^2\omega_x - \omega_x{}^2\omega)\text{Real}(q_5 q_{5x}) & 0 \\ (\omega^2 - \omega_x{}^2)\text{Real}(q_6 q_{6x}) & -(\omega^2\omega_x - \omega_x{}^2\omega)\text{Imag}(q_6 q_{6x}) & 0 \\ (\omega^2 - \omega_x{}^2)\text{Imag}(q_6 q_{6x}) & (\omega^2\omega_x - \omega_x{}^2\omega)\text{Real}(q_6 q_{6x}) & 0 \\ 0 & (\omega - \omega_x)\text{Imag}(q_5 q_{5x}) & (\omega^2 - \omega_x{}^2)\text{Real}(q_5 q_{5x}) \\ 0 & -(\omega - \omega_x)\text{Real}(q_5 q_{5x}) & (\omega^2 - \omega_x{}^2)\text{Imag}(q_5 q_{5x}) \\ 0 & (\omega - \omega_x)\text{Imag}(q_6 q_{6x}) & (\omega^2 - \omega_x{}^2)\text{Real}(q_6 q_{6x}) \\ 0 & -(\omega - \omega_x)\text{Real}(q_6 q_{6x}) & (\omega^2 - \omega_x{}^2)\text{Imag}(q_6 q_{6x}) \end{bmatrix}$$

$$A_1 = \text{Real}\left((k_{s1} + j\omega_x c_{s1})(y_{1x} - q_{5x})\omega^2 q_5 - (k_{s1} + j\omega c_{s1})(y_1 - q_5)\omega_x{}^2 q_{5x}\right)$$
$$B_1 = \text{Imag}\left((k_{s1} + j\omega_x c_{s1})(y_{1x} - q_{5x})\omega^2 q_5 - (k_{s1} + j\omega c_{s1})(y_1 - q_5)\omega_x{}^2 q_{5x}\right)$$
$$A_2 = \text{Real}\left((k_{s2} + j\omega_x c_{s2})(y_{2x} - q_{6x})\omega^2 q_6 - (k_{s2} + j\omega c_{s2})(y_2 - q_6)\omega_x{}^2 q_{6x}\right)$$
$$B_2 = \text{Imag}\left((k_{s2} + j\omega_x c_{s2})(y_{2x} - q_{6x})\omega^2 q_6 - (k_{s2} + j\omega c_{s2})(y_2 - q_6)\omega_x{}^2 q_{6x}\right)$$
$$C_1 = \text{Real}\left(k_{s1}[(y_{1x} - q_{5x})q_5 - (y_1 - q_5)q_{5x}] + jc_{s1}[(y_{1x} - q_{5x})\omega_x q_5 - (y_1 - q_5)\omega q_{5x}]\right)$$
$$D_1 = \text{Imag}\left(k_{s1}[(y_{1x} - q_{5x})q_5 - (y_1 - q_5)q_{5x}] + jc_{s1}[(y_{1x} - q_{5x})\omega_x q_5 - (y_1 - q_5)\omega q_{5x}]\right)$$
$$C_2 = \text{Real}\left(k_{s2}[(y_{2x} - q_{6x})q_6 - (y_2 - q_6)q_{6x}] + jc_{s2}[(y_{2x} - q_{6x})\omega_x q_6 - (y_2 - q_6)\omega q_{6x}]\right)$$
$$D_2 = \text{Imag}\left(k_{s2}[(y_{2x} - q_{6x})q_6 - (y_2 - q_6)q_{6x}] + jc_{s2}[(y_{2x} - q_{6x})\omega_x q_6 - (y_2 - q_6)\omega q_{6x}]\right)$$

2.4 MEASURING ROTOR VIBRATIONS

The identification of unbalanced forces and the parameters of the mountings and foundation from eq. (14) and eq. (30), require measurements of vibrations of the rotor and the machine structure. While measurements on the machine structure can be a straight forward business, measurements on the rotor needs further clarification. In some machines, the centre of gravity of the rotor may not be accessible for measurement while the machine is in operation because accessing the rotor may mean dismounting it. In this case, assuming the rotor is rigid, measurements of linear and rotational degrees-of-freedom at the rotor centre of gravity, $q_1(t)$ and $q_2(t)$ respectively, can be established by measurements on the rotor shaft at the bearings.

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$$q_1(t) = \frac{x_2}{x_1 + x_2} y_{r1}(t) + \frac{x_1}{x_1 + x_2} y_{r2}(t) \quad \text{and} \quad q_2(t) = \frac{y_{r1}(t) - y_{r2}(t)}{x_1 + x_2}$$

However, as the rotor is rotating, measurements $y_{r1}(t)$ and $y_{r2}(t)$ cannot be done using a contacting transducer. A non contacting transducer, *eg.* a proximity probe, have to be employed. For a rotor supported on rolling contact bearings, further simplification can be made by approximating vibrations of the rotor shaft at the bearings to those of the bearing housings. Thus:

$$q_1(t) = \frac{x_2}{x_1 + x_2} y_{B1}(t) + \frac{x_1}{x_1 + x_2} y_{B2}(t) \tag{31}$$

$$q_2(t) = \frac{y_{B1}(t) - y_{B2}(t)}{x_1 + x_2} \tag{32}$$

Where $y_{B1}(t)$ and $y_{B2}(t)$ are vibrations of the bearing housings, typically measured by a contacting transducer. This simplification may not apply to rotors with journal bearing supports due to the possibility of ample movement of the rotor shaft within the bearings.

3.0 NUMERICAL SIMULATION

The theory presented in Section 2 for the identification of unbalanced forces, mounting parameters and foundation parameters of rotating machines was verified by a numerical example.

EXAMPLE

A rotor of mass 50 kg is supported by two bearings at a span of 600 mm. The machine structure, excluding the rotor, is of mass 200 kg. The machine is supported by mountings also at a span of 600 mm. The foundation was simulated to be flexible, with an effective mass of 400 kg, effective stiffness of 4.0 MN/m and effective damping of 400 Ns/m. The machine system is shown in Fig. 2. Rotor unbalance equivalent to a centrifugal force of 1000 N and moment 500 Nm acting at rotor centre of gravity was assumed. The centre of gravity of the rotor is at mid-span between the bearings, while the centre of gravity of the machine structure is at the middle between the mounting locations. Complete data of the simulated system is shown in Table 1. The simulated data was used to compute simulated vibration responses of the machine and the foundation of a 6 DOF model like the one shown in Fig. 1, using MATLAB application package.

Table 1: Simulated Data of Rotating Machine

	Mass (kg)	Moment of Inertia (kgm²)	Stiffness (N/m)	Damping (Ns/m)	*Load (N)*
Rotor **(n = 1500 rpm)**	50	0.56	-	-	-
Bearing 1	-		0.50×10^6	100	-
Bearing 2	-		0.25×10^6	100	-
Unbalanced Force	-		-	-	***1000***
Unbalanced Moment	-		-	-	***500***
Machine structure	200	24	-	-	-
Support 1	-		2.0×10^6	200	-
Support 2	-		2.5×10^6	300	-
Foundation	400		4.0×10^6	400	-
Distances (mm)	$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 300$ mm				

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(i) Identification of Unbalanced Forces and Mounting Parameters

Vibration responses of the 6 DOF machine system excited by the centrifugal unbalanced force and unbalanced moment was computed. An unbalanced mass of Δm = 100g at a radial position of 200mm but 60° ahead of the unbalanced force, was simulated to be attached to the rotor at an axial distance x = 0.5 m from the rotor centre of gravity away from bearing 1. The unbalanced mass contributed to unbalanced force and equivalent unbalanced moment at the rotor centre of gravity, given by

$$\Delta F_u = \Delta m\omega^2 \times e \quad \text{and} \quad \Delta M_u = \Delta m\omega^2 \times ex,$$

where e is the radial position vector of the unbalanced mass. Vibration responses due to the combined presence of this unbalanced mass and the original unbalanced force and moment were computed. The unbalanced mass was then moved to a different angular position of 240° and new vibration responses computed. Table 2 shows the simulated vibration responses for the translational DOF of the system before and after adding unbalanced masses to the rotor.

Table 2: Simulated Vibration Responses Before and After Adding Mass to the Rotor

Degree-of-freedom	Response in mm (Δm = 0)	Response in mm (Δm = 200g at 60°)	Response in mm (Δm = 200g at 240°)
q_1	-1.394+j2.4388	-0.257+j1.1652	-2.0662+j1.2561
q_3	0.5673-j3.2189	-1.5075-j2.8047	1.8030-j2.1822
q_5	-1.3897+j5.8794	2.3305+j5.009	-3.5595+j3.8348
q_6	1.0046-j3.6899	-1.2435-j2.9575	2.3337-j2.3501
$y_1 = q_3 + x_5 q_4$	2.2314-j11.4761	-4.8945-j9.5055	6.5853-j7.6908
$y_2 = q_3 - x_6 q_4$	-1.0968+j5.0383	1.8796+j3.8961	-2.9794+j3.3264

The simulated response data in Table 2 and the system data in Table 1 were used to identify unbalanced force, moment and mounting parameters using eq. (14) and resulted in the following:

$$\begin{Bmatrix} k_{s1} + j\omega c_{s1} \\ k_{s2} + j\omega c_{s2} \\ F_u \\ M_u \end{Bmatrix} = \begin{Bmatrix} 2.0\text{x}10^3 + j314 \\ 2.5x10^3 + j471 \\ 1.0 \\ 0.5 \end{Bmatrix} x10^3$$

Hence

$k_{s1} = 2.0\text{x}10^6$ N/m, $k_{s2} = 2.5\text{x}10^6$ N/m
$c_{s1} = 200$ Ns/m, $c_{s2} = 300$ Ns/m
$F_u = 1000$ N, $M_u = 500$ N

Unbalanced force, moment and machine mounting parameters are exactly identified.

(ii) Identification of Foundation Parameters

The system was then simulated at a different rotor speed of 2500 rpm. Unbalanced force and moment are proportional to the square of the speed. Therefore, simulated unbalanced force and moment were proportionately adjusted to the new speed. Vibration responses due to the unbalances at the new speed were evaluated as shown in Table 3.

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Table 3: Simulated Vibration Responses at New Rotor Speed (n = 2500 rpm)

Degree-of-freedom	*Response in mm (Δm = 0)*
q_{1x}	0.0888-j0.0621
q_{3x}	-0.1319+j0.0219
q_{5x}	0.0364-j0.0024
q_{6x}	-0.0149-j0.0013
$y_{1x} = q_{3x} + x_5 q_{4x}$	-0.3887+j0.0384
$y_{2x} = q_{3x} - x_6 q_{4x}$	0.1249+j0.0055

The new and original vibrations, together with identified mounting parameters k_{s1}, k_{s2}, c_{s1} and c_{s2} were used to identify foundation parameters using eq. (30) and resulted in:

$$\begin{Bmatrix} k_f \\ c_f \\ m_f \end{Bmatrix} = \begin{Bmatrix} 4.0x10^6 \\ 400 \\ 400 \end{Bmatrix}$$

Therefore, the foundation parameters have been exactly identified. With the identified parameters, forces transmitted to the foundation may now be estimated as follows:

Force transmitted through mounting 1: $F_T = (k_f + j\omega c_f)(q_5 - y_1)$

Force transmitted through mounting 2: $F_T = (k_f + j\omega c_f)(q_6 - y_2)$

Forces transmitted to the foundation at the two speeds are shown in Table 4.

Table 4: Force Transmitted to The Foundation

Speed	Location	Transmitted Force (N)	Force Amplitude (N)
1500	Through mounting 1	-9,688+j27,684	29,331
	Through mounting 2	-612+j1,673	1,782
2500	Through mounting 1	852-j59	854
	Through mounting 2	-279-j21	280

4.0 CONCLUSIONS

A technique has been presented to identify unbalanced forces and foundation parameters of rotating machines with rigid rotors, from vibration measurements. The technique uses vibrations measured on a rotating rotor, using non contacting transducers, or on rotor bearing housings using contacting transducers, vibrations measured on the machine structure and the vibrations measured on the foundation.

The technique has been verified using simulated data for a two bearings rotor. The technique does not require prior knowledge of the unbalanced forces or bearing stiffness. While the technique has performed successfully with simulated data, further work is necessary in order to account for the real situation encountered in practical set-ups. Practical measurements are always contaminated with measurement errors, some of which are random but others may be systematic.

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