LOCALIZATION OF DAMAGE IN STRUCTURES BY ANALYTICAL MODEL IMPROVEMENT AND STRAIN ENERGY BALANCE

Noel G. Nalitolela
Department of Mechanical Engineering
University of Dar es Salaam
P. 0. Box 35131 Dar es Salaam, TANZANIA

ABSTRACT

System identification and mode shape correlation techniques are methods which have been proposed for localization of damage in structures. Their success, however, depends on the quality and completeness of the measured data and in many cases accurate localization is difficult. This paper present a new damage indicator utilizing the elements strain energies of the validated analytical models before and after the occurrence of damage. It is shown that for equality on the two sets of energy, stiffness of damaged elements have to reduce hence localized. The accuracy of localization depends on the ability of the validated models to reproduce modes of the structure rather than on the accuracy of the updated parameters. The method is demonstrated using simulated damage in beam elements.

NOMENCLATURE

U	strain energy of an element
U_{ref}	strain energy of an element per unit flexural rigidity
R	radius of curvature of deflection shape of an element
DI	damage indicator
CDI	cumulative damage indicator
EI	flexural rigidity of an element
$\{V^e\}$	mode shape deflection within an element
$[K^e]$	element stiffness matrix per unit flexural rigidity
$\frac{dy}{dx}$	slope of element deflection

1 INTRODUCTION

Vibration methods have in recent years been proposed as alternative technology for nondestructive testing (NDT) of structures. The simplest application of a vibration technique in NDT is the monitoring of changes in structural dynamic characteristics, particularly natural frequency, from the moment when the structure was first put into service. In this case a change in natural frequency indicates a structural change and often imply a damage has occurred. Alternatively in. quality control applications, variation in natural frequency from an average expected "normal" variation may also indicate improper structural integrity of a product [1]. The other more advanced application of vibration methods is not only the detection but also the localization of the damage in the structure.

A number of publications on damage localization methods exist [2-6]. The proposed methods are based on mode shape correlation, system identification /analytical model improvement and pattern recognition. With system identification/analytical model improvement the stiffness matrix of the validated models before and after the

occurrence of the damage are compared so as to identify regions of change. The success, however, depends on the accuracy of the system identification algorithm used which in most cases is deteriorated by measurement noise and incompleteness of the measured data. Mode shape correlation technique also suffer the problem of measurement noise and incompleteness in the measurement coordinates.

This paper present a new damage indicator. The analytical models before and after the occurrence of the damage are validated using measurement data before and after the damage is detected. Analytical mode shapes are then computed from the two validated models. The mode shapes are used to compute element strain energy before and after the occurrence of the damage. Using simulated damage on beam type elements, it is shown that for equality on the two sets of energy, stiffness of damaged elements in the damaged structure has to be reduced hence localized. The strain energy method is demonstrated using fixed-fixed beam and a plane frame with successful results. The localization accuracy depends only on the ability of the validated models to reproduce modes of the structure rather than on the physical meaning of the stiffness parameters.

2. THEORY

Consider a structure made of beam type elements. The strain energy in any element is a function of the second derivative of the elements deflection curve and the flexural rigidity.

$$U = 0.5 \int_{elem} EI \left(\frac{d^2y}{dx^2} \right)^2 dx$$

$$= 0.5 \int_{elem} EI \left(\frac{1}{R} \right)^2 dx \qquad (1)$$

where R is the radius of curvature.

Lets define a reference strain energy to be the energy per unit flexural rigidity. Thus, the reference strain energy of an element becomes:

$$U_{ref} = 0.5 \int_{elem} \left(\frac{d^2y}{dx^2} \right)^2 dx$$

$$= 0.5 \int_{elem} \left(\frac{1}{R} \right)^2 dx \qquad (2)$$

Consider a damage in an element where that damage is represented by a reduction in flexural rigidity. It is postulated that for a given mode of vibration the curvature of a beam element will increase if its stiffness is reduced. The increased curvature implying a reduction in R and therefore an increase in reference strain energy. Thus for a given element and a given jth mode of vibration, the following hold::

$$U^j_{ref(before)} - U^j_{ref(after)} < 0 \qquad (3)$$

The implication here is that when a damage takes place, there is a local stiffness reduction. For a given vibration mode, the damaged element tends to flex more than before the damage was inflicted. The other elements will have fairly the same degree of flexing as before the damage took place. However, inspection of equation (1) shows that comparing strain energy may not be very useful for damage localization, since the reduction of the denominator occur when there is a reduction in the numerator. Furthermore, the stiffness parameters of the elements before and after the ocurance of damage may not be known accurately.

Inspection of equation (2) suggest that the reference strain energy could be a more meaningful tool for damage localization provided the deflection shape of an element is known accurately. Thus we define Damage Indicator as

percentage change in reference strain energy:

$$DI = 100\left(\frac{U_{ref(before)} - U_{ref(after)}}{U_{ref(before)}}\right)$$

The damage indicators are summed over several modes in the frequency range of interest to define the Cumulative Damage Indicator (*CDI*).

$$CDI = \sum_{\text{mode i}}^{\text{mode n}} DI \qquad (5)$$

How do we determine the reference strain energy of the elements before and after damage has occurred? If we could accurately measure mode shapes of the structure when it was brought into service and some times later then the reference strain energy is computed by:

$$U_{ref} = 0.5 \{V^e\}^T [K^e] (V") \qquad (6)$$

In practice it is not feasible to determine the reference strain energy from measurement of mode shapes since the later are often incomplete and contaminated. The approach which has been found promising is to first formulate analytical model of the structure. Then perform test on the structure prior to the occurence of the damage and update the analytical model using test data. When a damage has occurred further tests are done on the structure and the original analytical model is updated using test data of the damaged structure. The updated/validated analytical models before and after damage has occurred are then used to compute mode shapes and hence evaluate reference strain energy using (6).

In the study reported in this paper the inverse eigenvalue sensitivity algorithm with virtual perturbation of the stiffness matrix [6], was used in analytical model improvement with good results. It would be of interest to find out the suitability of other system identification algorithms. The strain energy balance method is demonstrated by simulation examples.

3. SIMULATION EXAMPLES

3.1 EXAMPLE 1: A FIXED-FIXED BEAM

A beam of rigidity 5000 Nm^2, mass per unit length 3.5 kg/m and length 2.5 m was simulated with both ends fixed. Beam point receptances at 0.25 m and 1.0 m from one of the ends (nodes nl and n4 in figure 1) were simulated by modeling using a 10 elements FE model. The receptances cover the frequency range of 50 Hz to 250 Hz at 1 Hz interval and were contaminated by 5 percent mean zero random errors to simulate measurement noise. An initial analytical model was assumed with flexural rigidity 5300 Nm^2 and mass per unit length 3.3 kg/m. The contaminated receptances were used to update the initial parameters using a frequency domain inverse eigenvalue sensitivity algorithm with virtual perturbation of the stiffness matrix [6]. The updating was performed by adjusting the parameters of the ten elements independently and result in a validated model of the undamaged beam.

A damage was then simulated in element 9 (between nodes n8 and n9) by a 20 % reduction in flexural rigidity. The point receptances at nodes nl and n4 were then simulated using exact flexural and mass parameters. The receptances were also contaminated by 5 % random errors and used to update the initially assumed theoretical parameters.

Table 1 shows the parameters of the validated models of the beam before and after simulation of the damage. In both updating cases contaminated transfer receptances at nodes n6 and n8 in

addition to nl and n2 were also simulated for excitations at nl and n4. These were used as control measurements for "mode shape" correlation purposes as required by the updating algorithm.

Figure 2 shows a plot of cumulative damage indicator for the first 6 modes, using exact modes of the undamaged and damaged beam. The damage is perfectly localized by a high negative value of CDI for element 9.

Figure 3 is a plot of cumulative damage indicator using modes of the models validated using simulated noisy FRF data. Again the damage in the 9th element has been successfully localized.

Figure 4 is a plot of percentage difference of stiffness parameters of the models validated after and before the occurrence of the damage. It can be seen that localization is poor. System identification on its own is not reliable due to inaccuracy in the updated parameters.

3.2 EXAMPLE 2: A PLANE FRAME

A plane frame made of 5 beam members, B1 to B5, was simulated by a 14 elements 39 DOF model (figure 5). The frame was simulated to be supported by soft springs as shown. The simulated stiffness (Nm^2) and mass (kg/m) parameters were as follow:

Parameters of simulated Frame

EI	21333	5208	14292	9000	9000
M	6.280	3.925	5.495	4.710	4.710

Point receptances at midspan of beams B4 and B5 were computed at 0.5 Hz interval in the frequency range of 50 Hz to 150 Hz. The receptances were contaminated by 5 % mean zero random errors to simulate measurement noise.

Furthermore, an initial analytical model was assumed with the following parameters:

Parameters of the initial analytical model

EI	25000	6000	10000	11000	9000
M	6.0	3.5	5.0	5.5	5.5

The initial model parameters were updated using the contaminated point receptances, to obtain a validated model of the frame before damage.

To simulate a structural damage, the node at mid span of beam member B3 was remodeled into a pinned connection, giving common horizontal and vertical degrees of freedom for the two elements joined at this point but separate rotational degrees of freedom. The frame now had 40 DOF after the damage. Using the simulated parameters, point receptances of the 40 DOF frame were computed at the mid span of beams B4 and B5. These were also contaminated by 5 % mean zero random errors to simulate measurement noise. The simulated receptances were then used to update the initial analytical model to obtain a validated model after the ocurance of the damage. In the updating the initial model structure with 39 DOF was maintained although the data belonged to a 40 DOF system.

Figure 6 show the contaminated simulated receptances before and after the damage. The updated parameters before and after damage are given in table 2. The updating was performed using inverse eigenvalue sensitivity algorithm with virtual perturbation of the stiffness matrix. In both updating cases transfer receptances at the FE nodes of beams B1 and B5 were also simulated with random errors. These were taken as control measurements for "mode shape" correlation purposes.

Figure 7 show a plot of cumulative damage indicator (CDI) whereas figure 8 is a plot of damage indication by simply taking the difference in the validated stiffness parameters.

It can be seen that where analytical model improvement failed to localize the damage due inaccuracy in the parameters and particularly made difficult due to incorrect model structure, the strain energy method using cumulative damage indicator has successfully localized the damage to beam B3.

4. CONCLUSION

System identification or analytical model improvement alone is not reliable for damage localization. A new damage indicator utilizing reference strain energy of beam type elements (or macro elements) has been presented. Simulation results show this indicator to be a very effective tool for damage localization in beam type structures. The success of the localization depends on the ability of the validated models to reproduce modes of the structure rather than on the accuracy of the validated parameters. Further work however is required to assess its capability and limitations in real structure damage.

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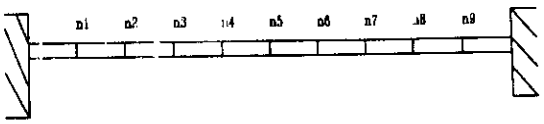


Figure 1. Fixed - fixed beam

Element No.	1	2	3	4	5	6	7	8	9	10
EI before damage	5062	5037	5029	4977	4999	5010	5026	5032	5010	4948
EI After damage	5099	4936	5043	4931	4904	4911	4926	5128	4745	4934

Table 1. Validated stiffness parameter al beam before and after damage

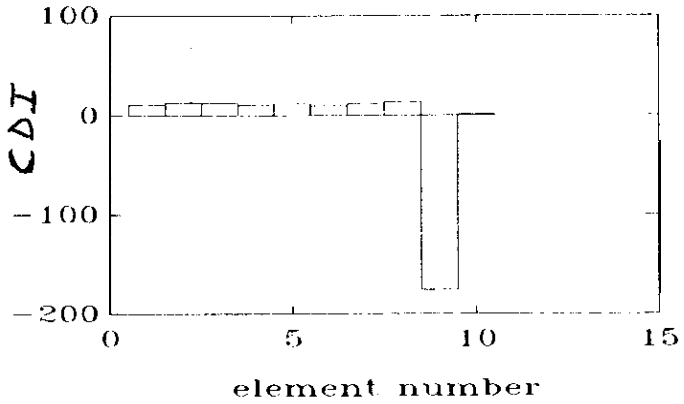


Figure 2. Demage indication for beam; exact modeshapes

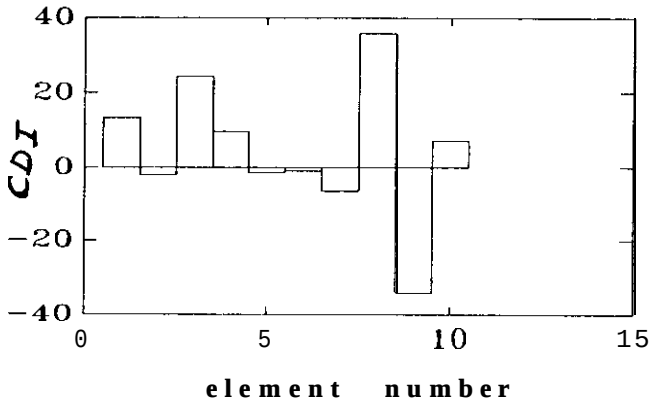
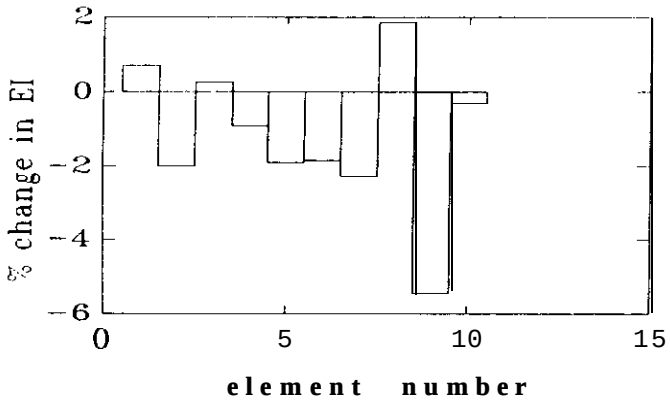


Figure 3. Damege indicator for beau: Using modes of validated models



Awe 4: Localicatica by % change in beam stiffness parameters

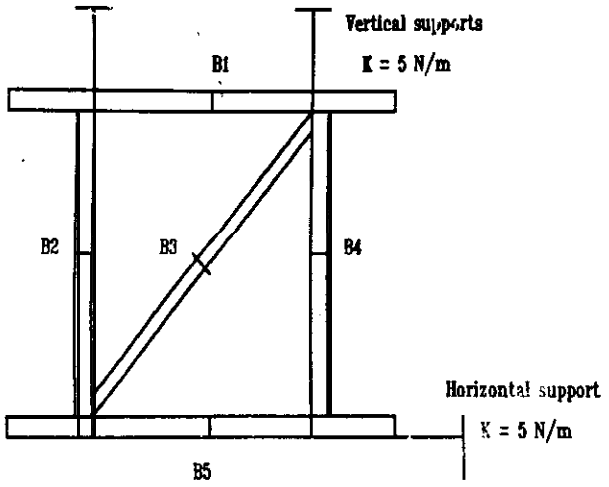


Figure 5. Plane frame with 14 beam elements

UPDATED PARAMETERS BEFORE SIMULATION OF DAMAGE					
	B1	B2	B3	B4	B5
EI (Nm^2)	21883	3283	14491	9096	9119
M (kg/m)	6.386	3.982	5.570	4.739	4.739
UPDATED PARAMETERS AFTER SIMULATION OF DAMAGE					
EI (Nm^2)	21043	5277	14294	8978	8977
M (kg/m)	6.225	3.939	5.493	4.679	4.699

Table 2. Validated parameters of frame before and after damage

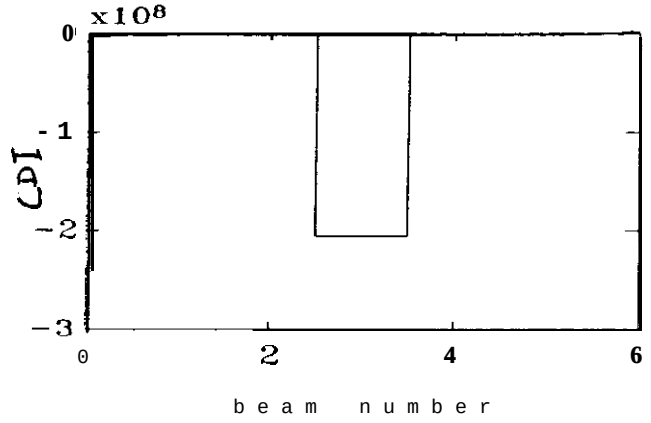


Figure 7: Damage indicator for the frame

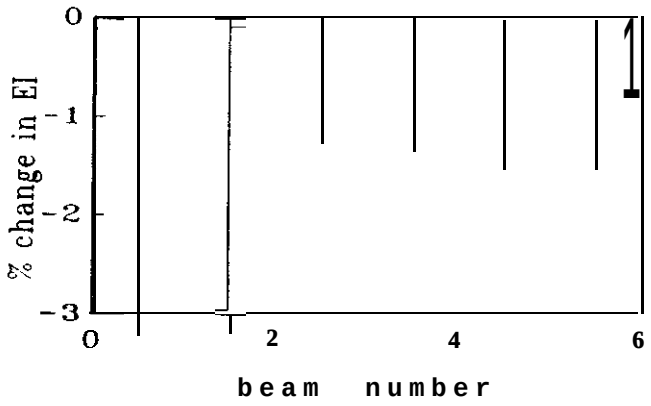


Figure 8: Localization by % change of frame stiffness parameters

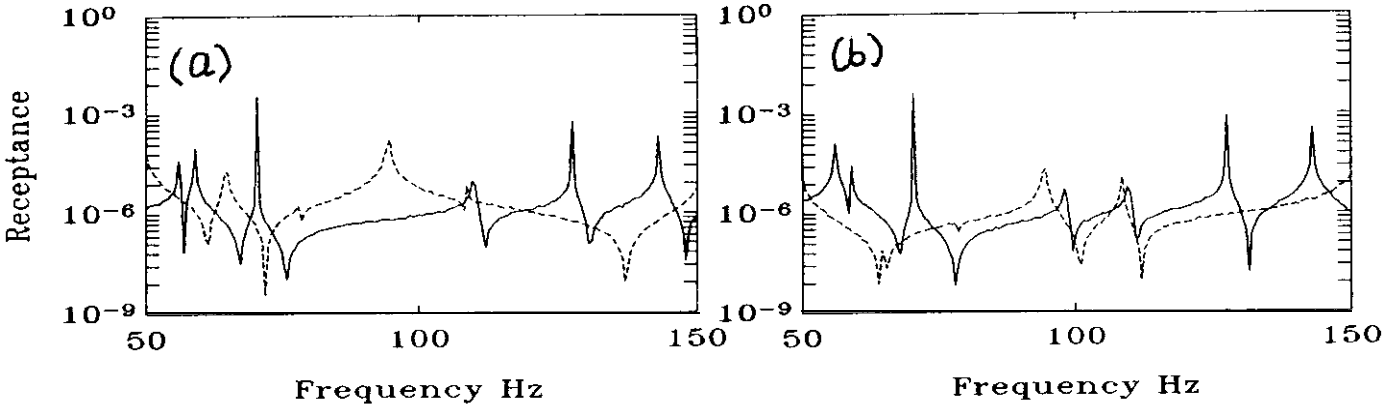


Figure 6: Simulated receptance of frame before and after damage a) at mid of B4 b) at mid of B5